Fall 2006 **CMSC 451: Homework 9** William Gasarch

Due Monday Nov 27, at Beginning of class

IMPORTANT NOTE: Be CLEAR, Be SHORT. You may lose points for a correct answer if the TA does not understand it.

1. (0 points). Write your name clearly. Staple your HW. READ Chap on Dynamic programming (the parts we did in class) AND your classnotes (NOTE- some of what we did in class is not in the text). NOTE- make sure to read up on MEMOIZATION in the book. Also, when and where are the final?

2. (30 points) Give an example of an instance of WEIGHTED INTERVAL SCHEDULING where the input has 20 intervals and where the greedy approach that was used for INTERVAL SCHEDULING does not work. In particular we want to see $100 \times$ GREEDYSOLUTION $<$ OPTIMAL SOLUTION.

3. (30 points) Consider the following problem:

   INPUT: $(w_1, \ldots, w_n, W)$

   OUTPUT: a set $S \subseteq \{w_1, \ldots, w_n\}$ such that $|(\sum_{w \in S} w) - W|$ is as small as possible. (NOTE- we do NOT insist that the sum be $\le W$.)

   (a) Write a recurrence that is helpful to solve this problem.

   (b) Write a DYNAMIC PROGRAM for this problem. Give its run time in O-notation

   (c) Write a RECURSIVE PROGRAM for this problem that has a similar runtime.

TURN OVER

4. (40 points) Consider the following game. Initially there are three piles of stones of sizes a, b, c which we denote (a, b, c). We denote the piles by I, II and III. When it is a players turn to move he or she can do exactly one of the following

   - Remove 1, 2, or 3 stones from pile I. (NOTE- If there are 2 stones in pile I and you take this option then you can only remove 1 or 2 stones. If there is 1 stone in pile I and you take this option then you have to remove the 1 stone. If there are 0 stones in pile I then you cannot take this option.)
   - Remove 1,3, or 5 stones from pile II. (Similar comments apply here as in the first option.)
   - Remove a prime number of stones from pile III. (NOTE- If there are 0 or 1 stones in pile III then this move is not an option. If there are $m \geq 2$ stones in pile III then you can remove any prime p number of stones such that $2 \leq p \leq m$.)

   If a player cannot move then he or she loses (and the other player wins). The players take turns, but ALICE goes first and BOB goes second. Write psuedocode for a program that does the following: Given (a, b, c) $(a, b, c \in N$, and they could be 0) determine who has a winning strategy, ALICE or BOB. The program has to run in time $O(abc)$.