Fall 2006 **CMSC 451: Homework 10** William Gasarch

Due Monday Dec 4, at Beginning of class

IMPORTANT NOTE: Be CLEAR, Be SHORT. You may lose points for a correct answer if the TA does not understand it.

1. (0 points). Write your name clearly. Staple your HW. READ Chap on NPC (the parts we did in class) AND your classnotes (NOTE- some of what we did in class is not in the text). When and where are the final?

2. (40 points) Let

$$HC = \{G \mid \ G \text{ has a Hamiltonian Cycle } \}.$$

   Let

$$HP = \{(G, a, b) \mid \ G \text{ has a Hamitonian Path from } a \text{ to } b \ \}.$$

   (a) Show that $HP \le_m HC$.

   (b) Show that if $HP \in P$ then $HC \in P$. (You can show this by showing that $HC \le_m HP$; however, if this is difficult there is another way to do it.)

3. (30 points) Show that 2-SAT is in P. (HINT- first take the formula and make a graph of it as follows: Every literal is a vertex, and for each clause $(x \vee y)$ put in the edge $(x, \overline{y})$ and $(\overline{x}, y)$.)

4. (30 points).

   (a) Find an algorithm that, given a number n, factors it in $O(\sqrt{n})$ steps.

   (b) Why does the first part of this problem NOT imply that factoring can be done efficiently?

5. (Extra Credit- but THINK about it, I want to go over it when HW is due) If L is a subset of $\{0,1\}^*$ then

   $L^0 = \{\epsilon\}$ (the empty string).

   $L^1 = L$

   $L^{i+1} = \{w \mid (\exists x, y)[w = xy \wedge x \in L \wedge y \in L^i\}$.

   $L^* = \bigcup_{i=0}^{\infty} L^i$

   Show that if $L \in P$ then $L^* \in P$.