Fall 2006 **CMSC 451: Homework 11** William Gasarch

Due Monday Dec 11

IMPORTANT NOTE: Be CLEAR, Be SHORT. You may lose points for a correct answer if the TA does not understand it.

1. (0 points). Write your name clearly. Staple your HW. READ Chap on NPC (the parts we did in class) AND your classnotes (NOTE- some of what we did in class is not in the text). When and where are the final?

2. (30 points) Let DNF-SAT be the set of all formulas of the form

   $$C_1 \vee C_2 \vee C_3 \vee \cdots \vee C_k$$

   where each C_i is of the form $(L_1 \wedge L_2 \wedge \cdots \wedge L_m)$ (where each L_i is either a variable or the negation of a variable.)

   Show that DNF-SAT is in P.

3. (30 points) Give an algorithm that does the following in $O(n^2)$ steps: Given a graph G,

   - Output NO if it does not have a vertex cover of size 17.
   - Output YES and also an actual Vertex cover of size 17 if it has one. (NOTE- in class out algorithm only output YES or NO, here we also WANT the actual VC.)

4. (40 points) Recall that a graph is PLANAR if you can draw it in the plane without two edges crossing.

   FACTS YOU CAN USE FOR THIS PROBLEM:

   - If a graph is planar and you remove any edge or vertex, the remaining graph is planar.
   - If a graph is planar then there exists a vertex of degree ≤ 5.
   - Every Planar graph is 6-colorable. That is, there is a way of assigning to every vertex a color so that no two adjacent vertices have the same color, and at most 6 colors are used. (It is actually know that every planar graph is 4-colorable but that will not be needed for this problem.)

   Show that the following can be done in polynomial time: given a planar graph, find a 6-coloring of it.

   HINT1: The algorithm is recursive.

   HINT2: The algorithm first finds the vertex of degree ≤ 6.