

CMSC 132: Object-Oriented Programming II



Advanced Tree Structures

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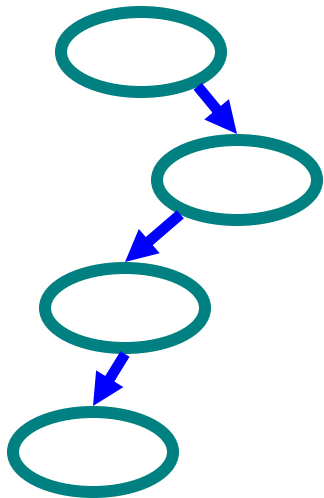
Overview

- **Binary trees**
 - **Balance**
 - **Rotation**
- **Multi-way trees**
 - **Search**
 - **Insert**
- **Indexed tries**

Tree Balance

■ Degenerate

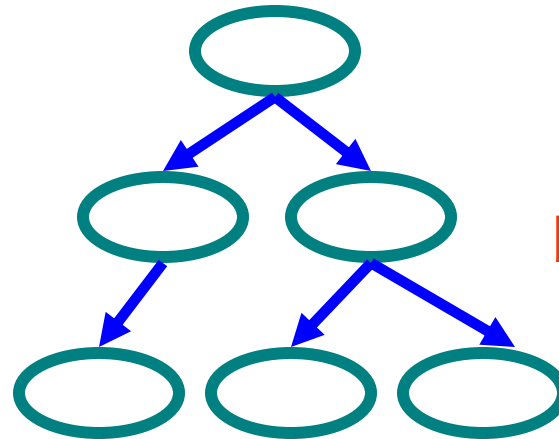
- Worst case
- Search in $O(n)$ time



**Degenerate
binary tree**

■ Balanced

- Average case
- Search in $O(\log(n))$ time



**Balanced
binary tree**

Tree Balance

■ Question

- Can we keep tree (mostly) balanced?

■ Self-balancing binary search trees

- AVL trees
- Red-black trees

■ Approach

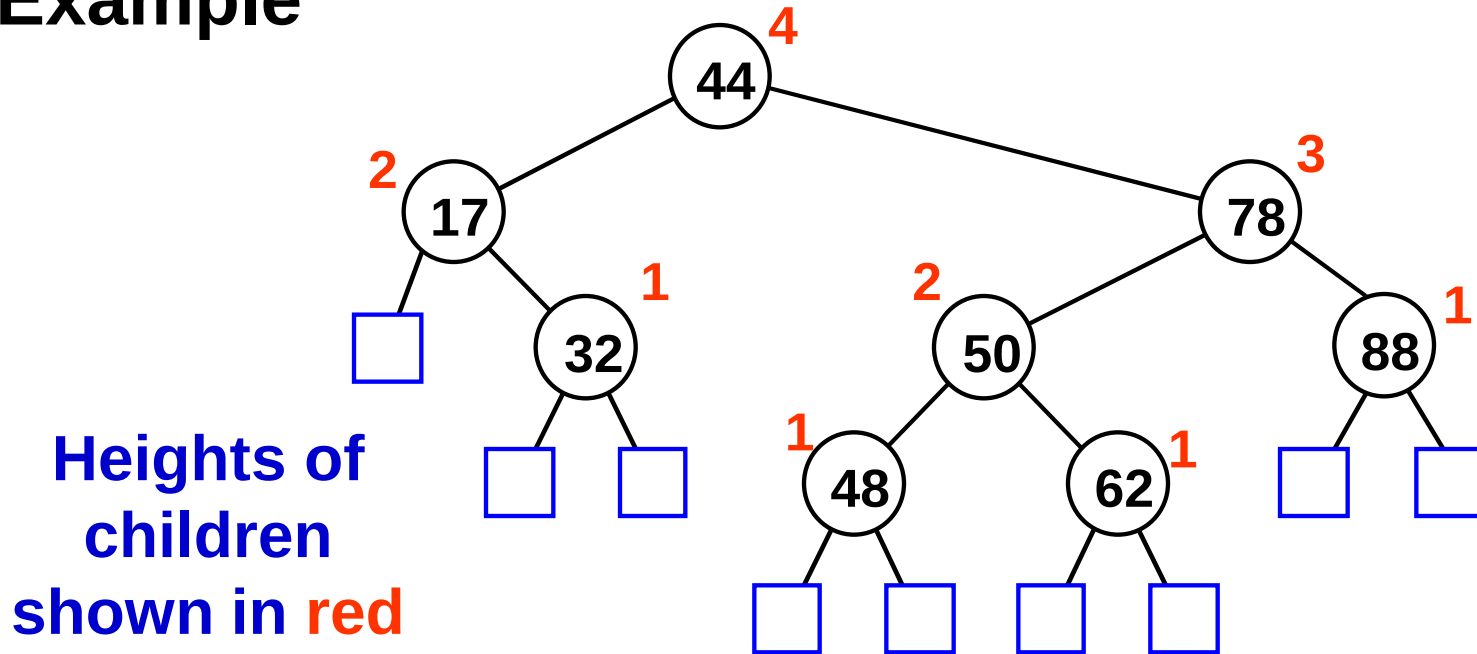
- Select invariant (that keeps tree balanced)
- Fix tree after each insertion / deletion
 - Maintain invariant using **rotations**
- Provides operations with $O(\log(n))$ worst case

AVL Trees

■ Properties

- Binary search tree
- Heights of children for node **differ by at most 1**

■ Example



AVL Trees

■ History

- Discovered in 1962 by two Russian mathematicians, Adelson-Velskii & Landis

■ Algorithm

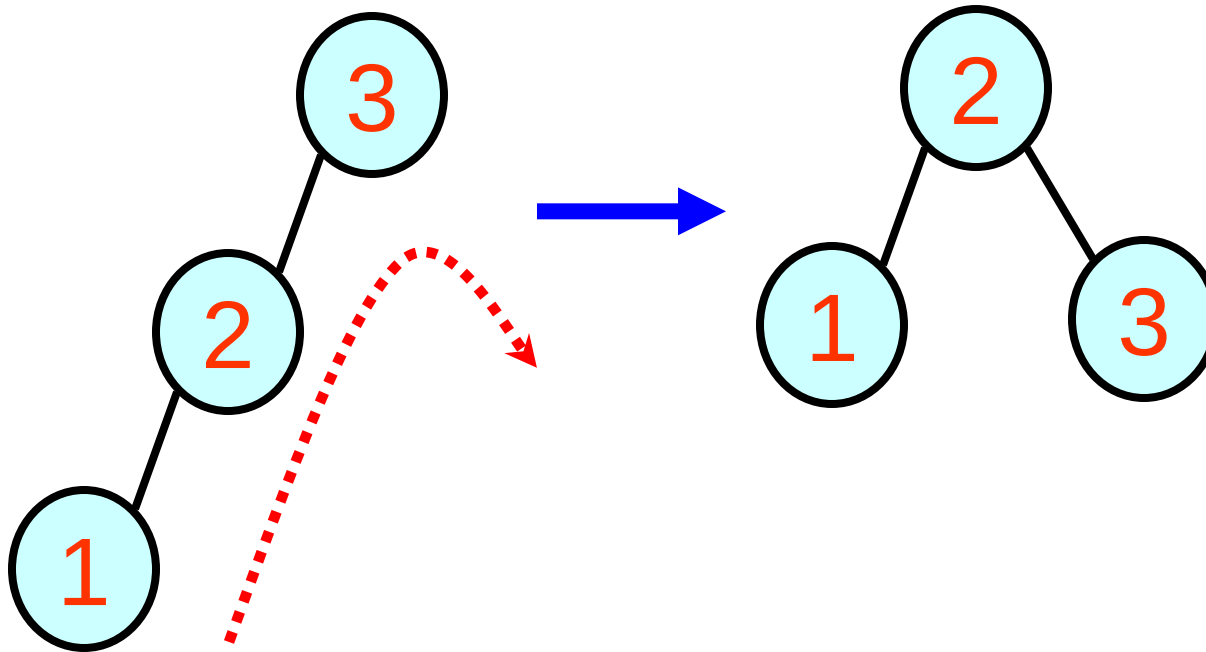
1. Find / insert / delete as a binary search tree
2. After each insertion / deletion
 - a) If height of children differ by more than 1
 - b) **Rotate** children until subtrees are balanced
 - c) Repeat check for parent (until root reached)

Tree Rotations

- **Changes shape of tree**
 - **Rotation moves one node up in the tree and one node down**
 - **Height is decreased by moving larger subtrees up and smaller subtrees down**
- **Types**
 - **Single rotation**
 - **Left**
 - **Right**
 - **Double rotation**
 - **Left-right**
 - **Right-left**

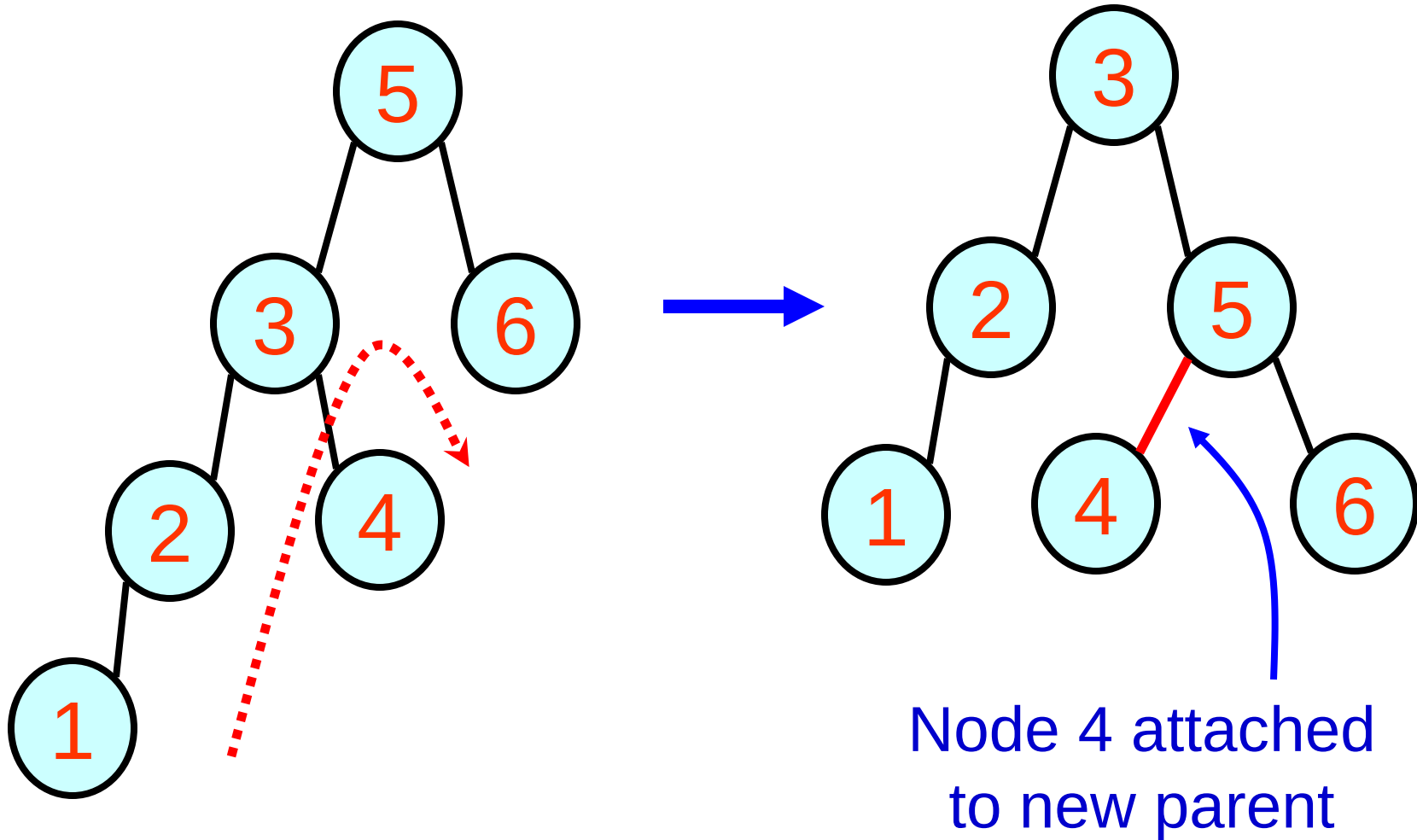
Tree Rotation Example

■ Single right rotation

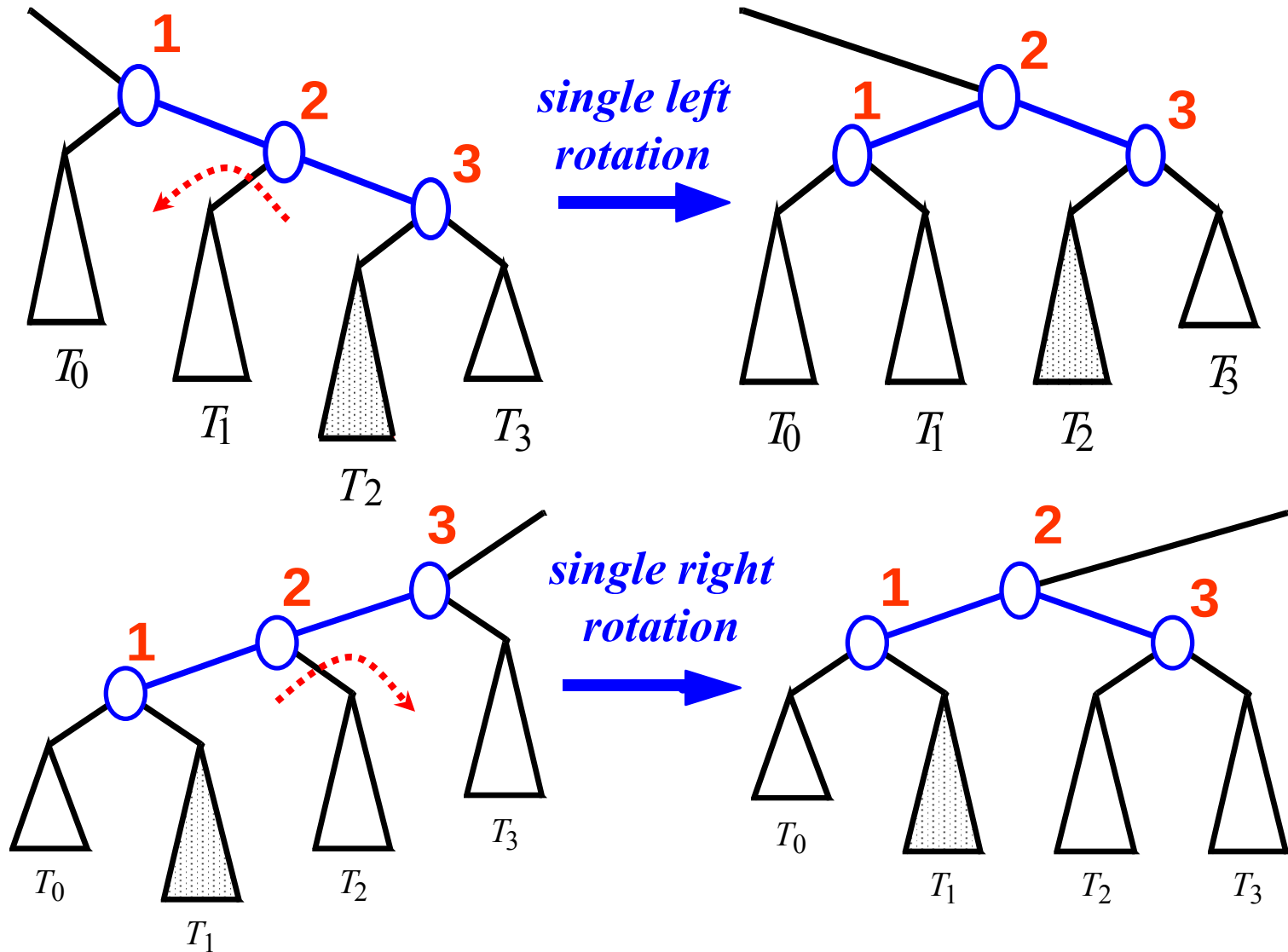


Tree Rotation Example

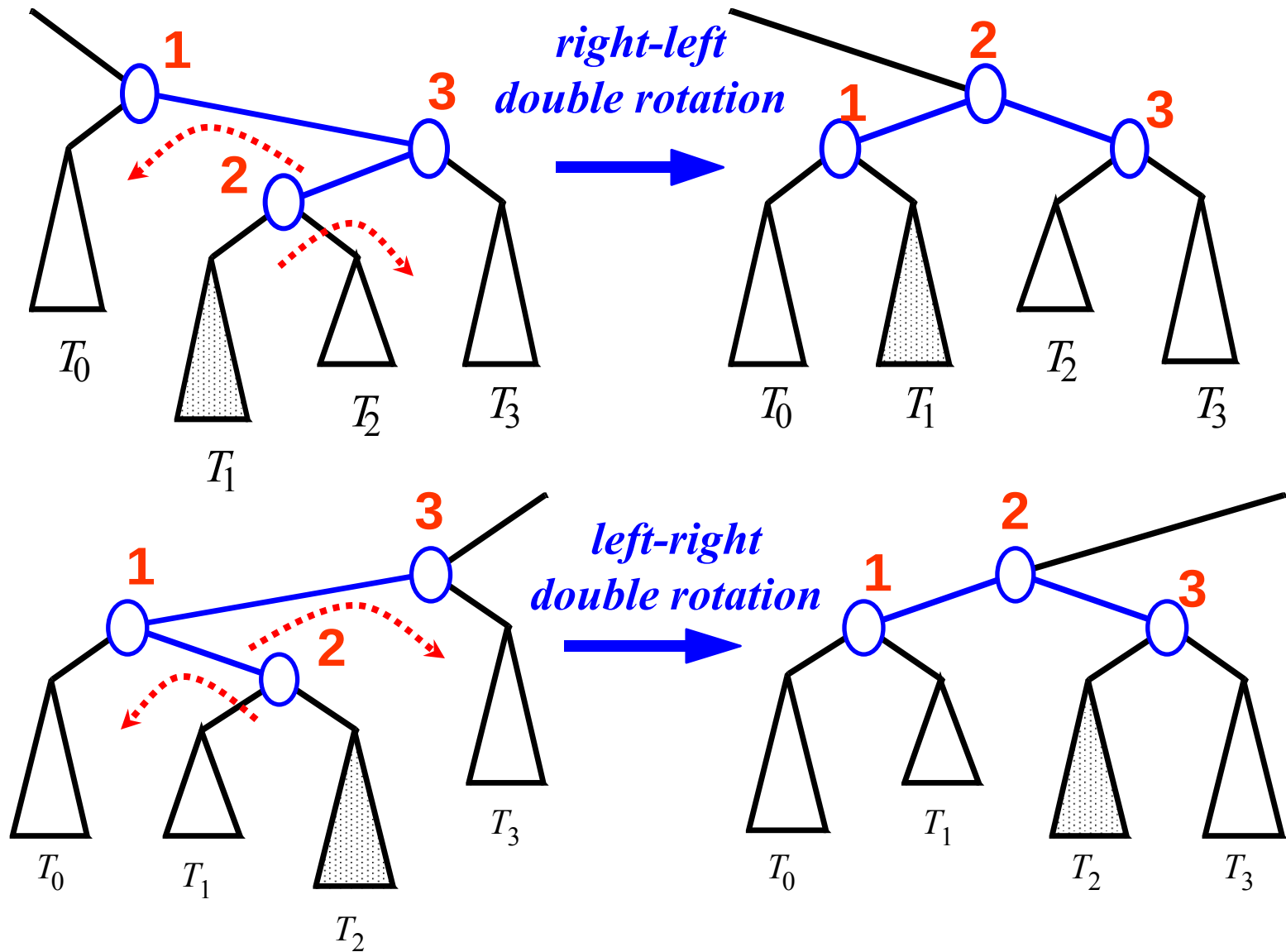
■ Single right rotation



Example – Single Rotations



Example – Double Rotations



Red-black Trees

■ Properties

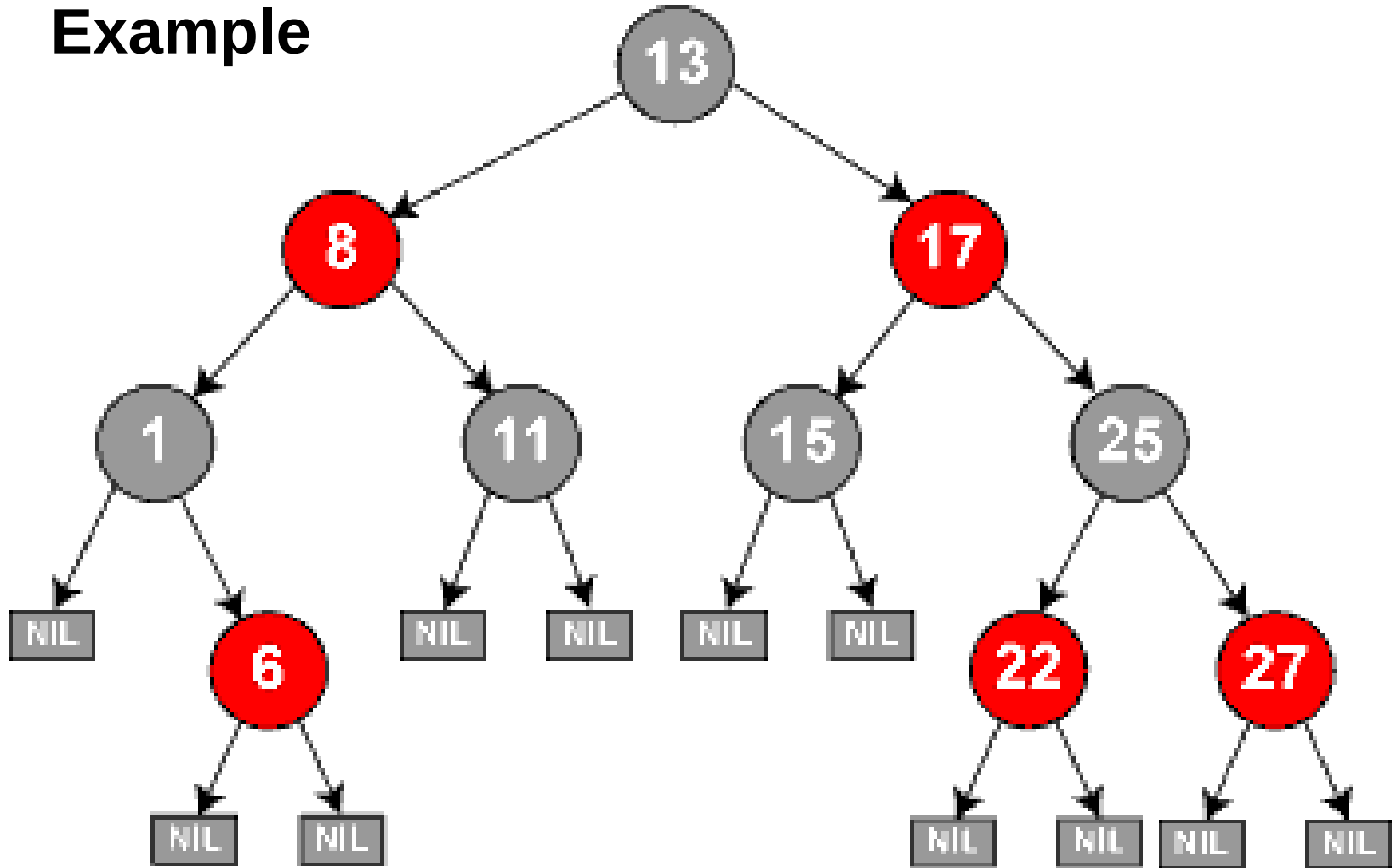
- Binary search tree
- Every node is **red** or **black**
- The root is black
- Every leaf is black
- All children of red nodes are black
- For each leaf, same # of black nodes on path to root

■ Characteristics

- Properties ensures no leaf is twice as far from root as another leaf

Red-black Trees

■ Example



Red-black Trees

■ History

- Discovered in 1972 by Rudolf Bayer

■ Algorithm

- Insert / delete may require complicated bookkeeping & rotations

■ Java collections

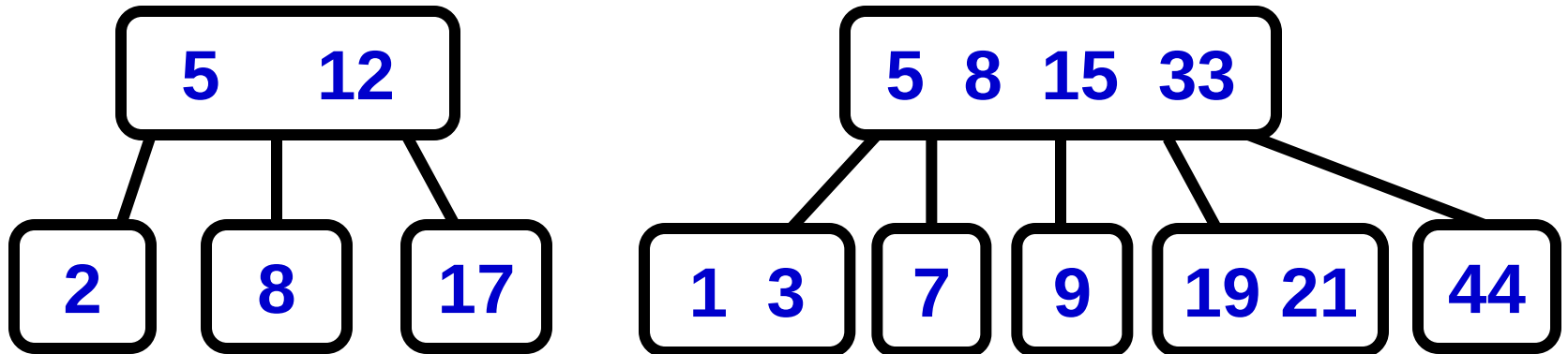
- TreeMap, TreeSet use red-black trees

Multi-way Search Trees

■ Properties

- Generalization of binary search tree
- Node contains $1 \dots k$ keys (in sorted order)
- Node contains $2 \dots k+1$ children
- Keys in j^{th} child $< j^{\text{th}}$ key $<$ keys in $(j+1)^{\text{th}}$ child

■ Examples



Types of Multi-way Search Trees

■ 2-3 tree

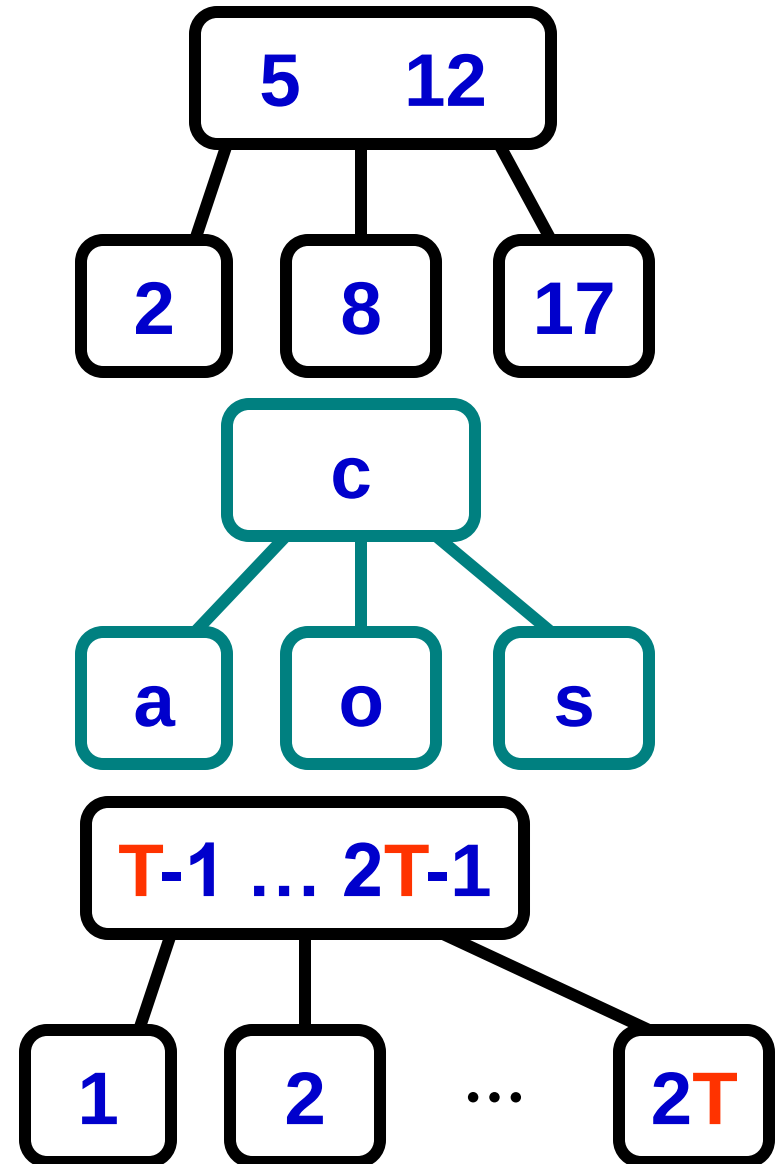
- Internal nodes have 2 or 3 children

■ Index search trie

- Internal nodes have up to 26 children (for strings)

■ B-tree

- T = minimum degree
- Non-root internal nodes have $T-1$ to $2T-1$ children
- All leaves have same depth



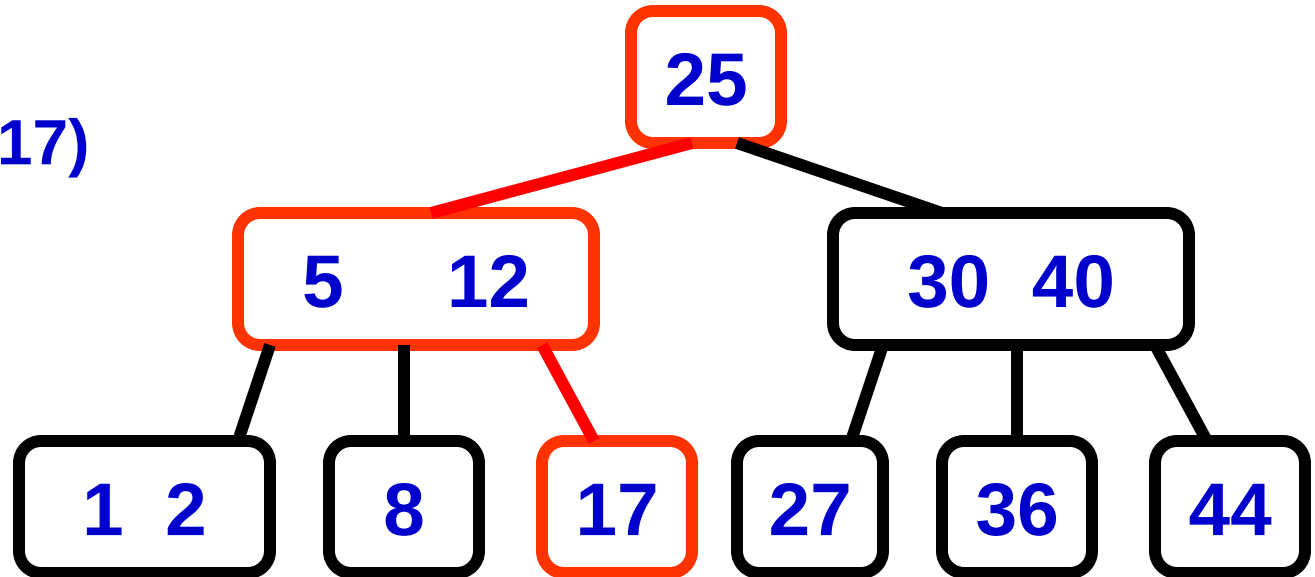
Multi-way Search Trees

■ Search algorithm

1. Compare key **x** to 1...k keys in node
2. If $x = \text{some key}$ then return node
3. Else if ($x < \text{key } j$) search child **j**
4. Else if ($x > \text{all keys}$) search child **k+1**

■ Example

■ Search(17)



Multi-way Search Trees

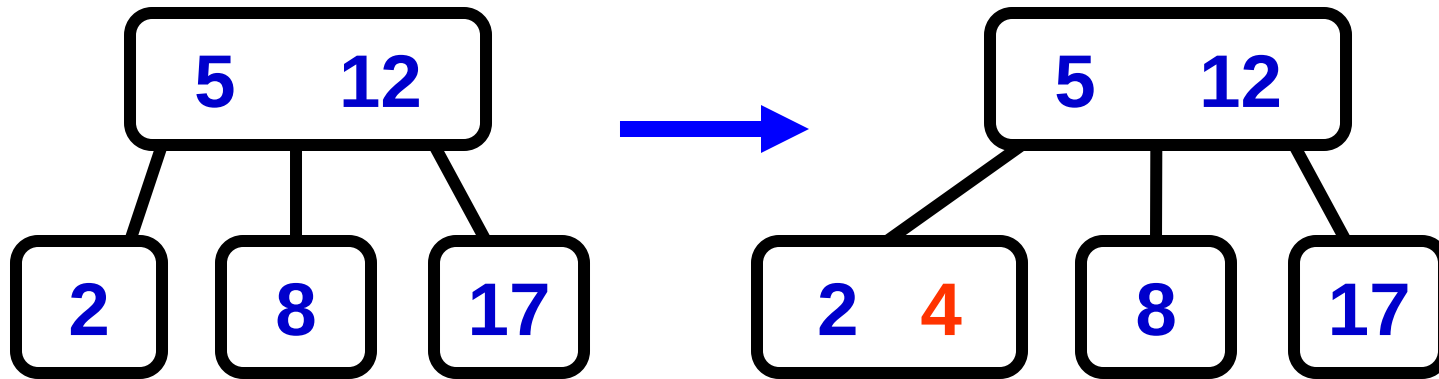
■ Insert algorithm

1. Search key **x** to find node **n**
2. If (**n** not full) insert **x** in **n**
3. Else if (**n** is full)
 - a) Split **n** into two nodes
 - b) Move middle key from **n** to **n**'s parent
 - c) Insert **x** in **n**
 - d) Recursively split **n**'s parent(s) if necessary

Multi-way Search Trees

■ Insert Example (for 2-3 tree)

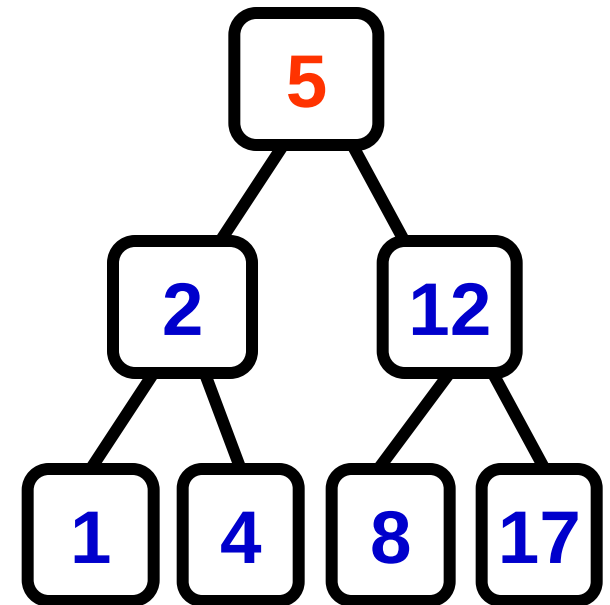
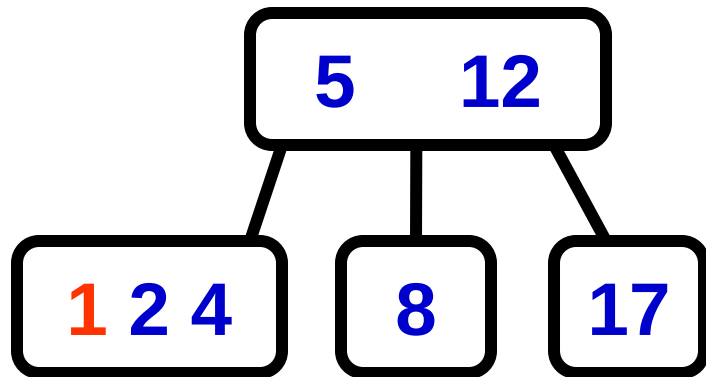
■ Insert(4)



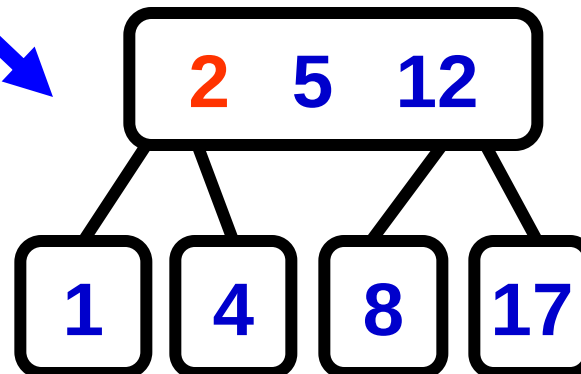
Multi-way Search Trees

■ Insert Example (for 2-3 tree)

■ Insert(1)



Split node



Split parent

B-Trees

■ Characteristics

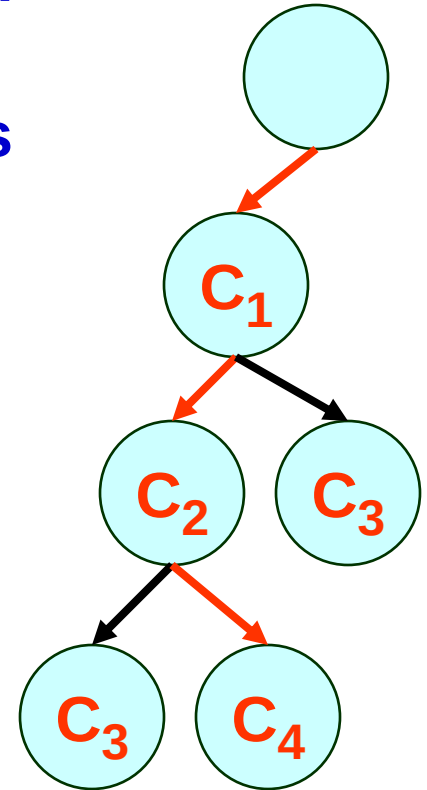
- Height of tree is $O(\log_T(n))$
- Reduces number of nodes accessed
- Wasted space for non-full nodes

■ Popular for large databases

- 1 node = 1 disk block
- Reduces number of disk blocks read

Indexed Search Tree (Trie)

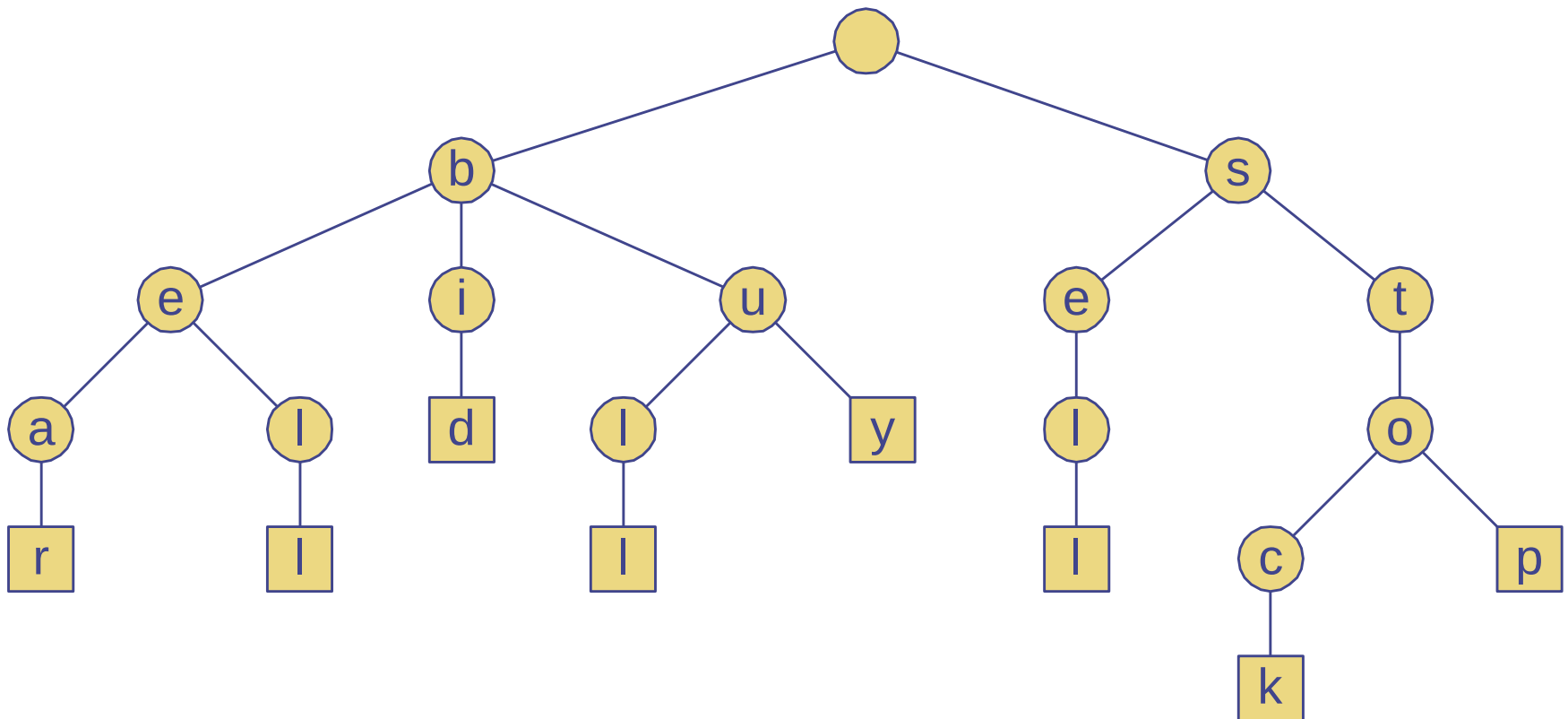
- Special case of tree
- Applicable when
 - Key **C** can be decomposed into a sequence of subkeys **C₁, C₂, ... C_n**
 - Redundancy exists between subkeys
- Approach
 - Store subkey at each node
 - Path through trie yields full key



Standard Trie Example

■ For strings

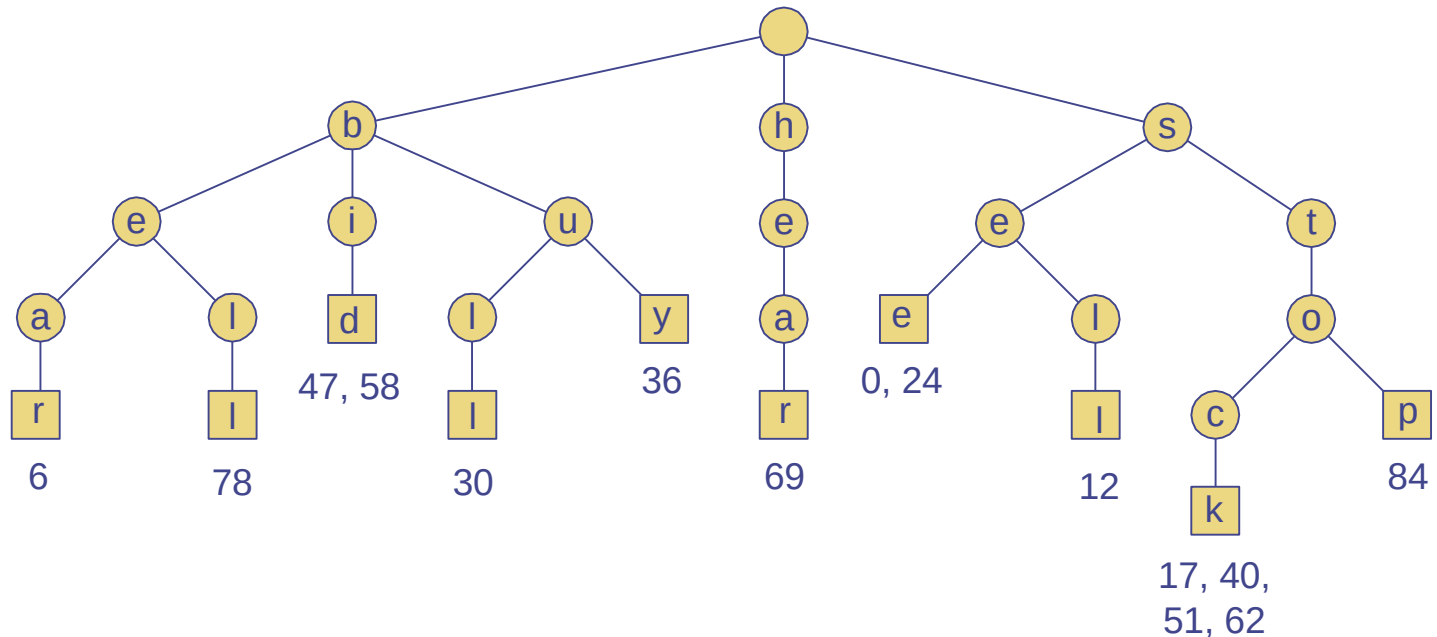
■ { bear, bell, bid, bull, buy, sell, stock, stop }



Word Matching Trie

- Insert words into trie
- Each leaf stores occurrences of word in the text

s	e	e		a		b	e	a	r	?		s	e	l	l		s	t	o	c	k	!	
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
s	e	e		a		b	u	l	l	?		b	u	y		s	t	o	c	k	!		
24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	
b	i	d		s	t	o	c	k	!		b	i	d		s	t	o	c	k	!			
47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68		
h	e	a	r		t	h	e		b	e	l	l	?		s	t	o	p	!				
69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88				



Compressed Trie

■ Observation

- Internal node v of T is redundant if v has one child and is not the root

■ Approach

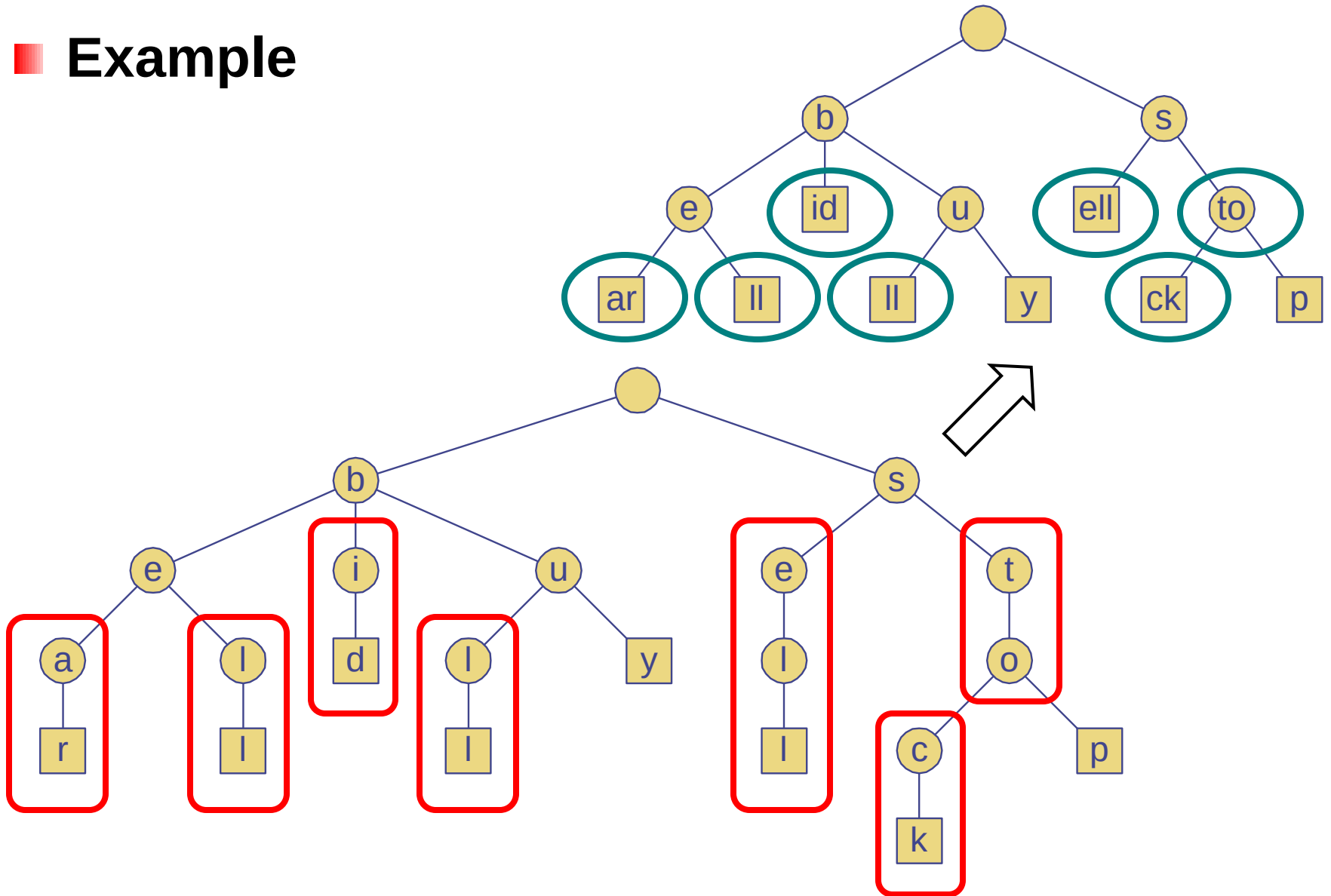
- A chain of redundant nodes can be compressed
 - Replace chain with single node
 - Include concatenation of labels from chain

■ Result

- Internal nodes have at least 2 children
- Some nodes have multiple characters

Compressed Trie

■ Example



Tries and Web Search Engines

- **Search engine index**
 - Collection of all searchable words
 - Stored in compressed trie
- **Each leaf of trie**
 - Associated with a word
 - List of pages (URLs) containing that word
 - Called occurrence list
- **Trie is kept in memory (fast)**
- **Occurrence lists kept in external memory**
 - Ranked by relevance