CMSC 132: OBJECT-ORIENTED PROGRAMMING II

Advanced Tree Structures

Department of Computer Science

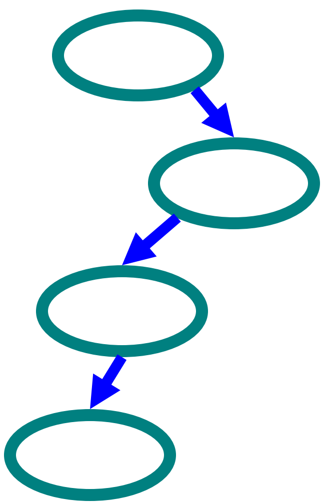
University of Maryland, College Park

Overview

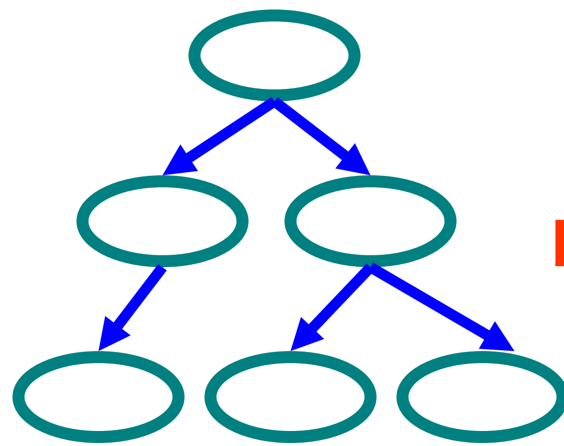
- Binary trees
  - Balance
  - Rotation
- Multi-way trees
  - Search
  - Insert
- Indexed tries

Tree Balance

- Degenerate
  - Worst case
  - Search in O(n) time
- Balanced
  - Average case
  - Search in O(log(n)) time



Degenerate binary tree



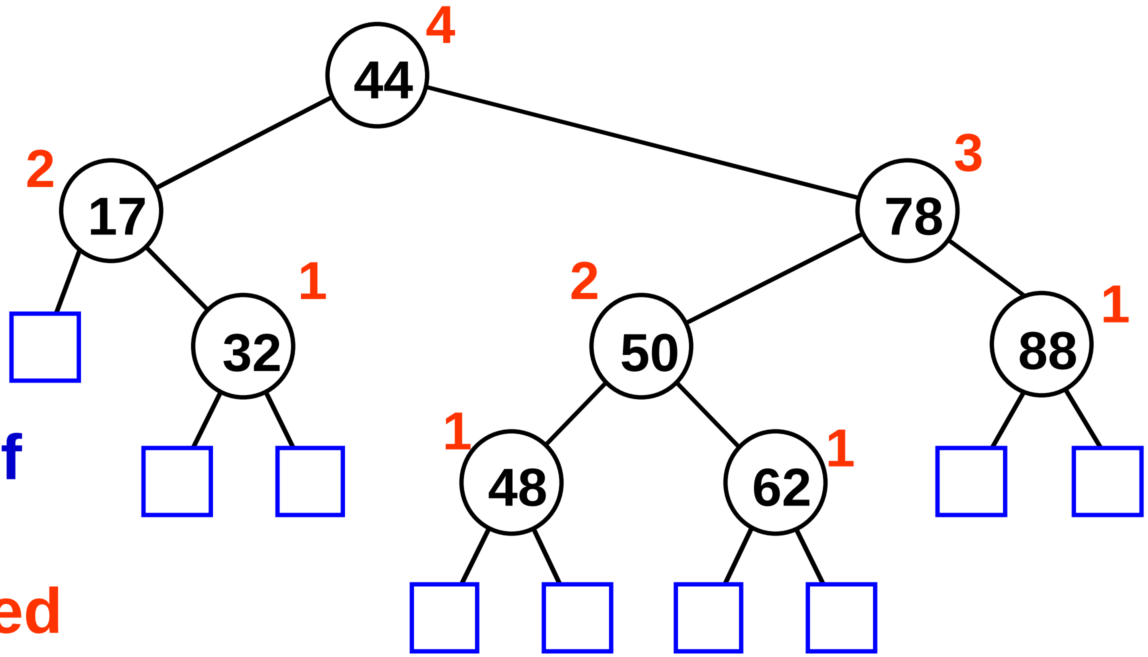
Balanced binary tree

Tree Balance

- **Question**
  - Can we keep tree (mostly) balanced?
- **Self-balancing binary search trees**
  - AVL trees
  - Red-black trees
- **Approach**
  - Select invariant (that keeps tree balanced)
  - Fix tree after each insertion / deletion
    - Maintain invariant using rotations
  - Provides operations with O(log(n)) worst case

AVL Trees

- **Properties**
  - Binary search tree
  - Heights of children for node differ by at most 1
- **Example**



Heights of children shown in red

AVL Trees

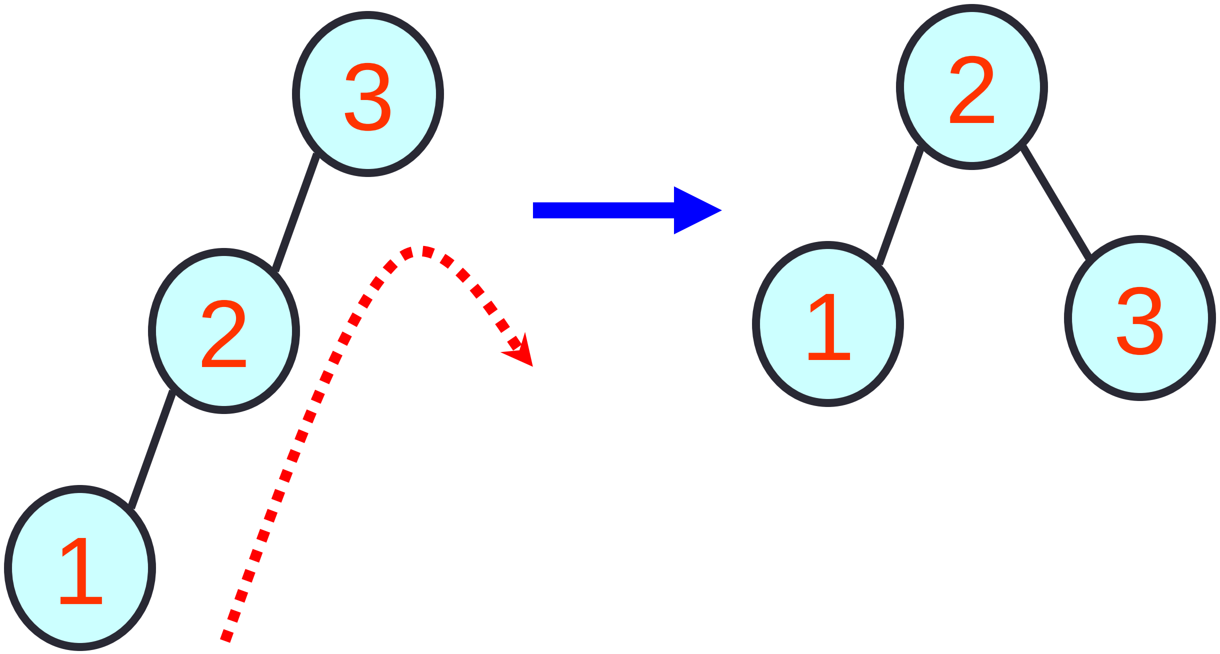
- **History**
  - Discovered in 1962 by two Russian mathematicians, Adelson-Velskii & Landis
- **Algorithm**
  1. Find / insert / delete as a binary search tree
  2. After each insertion / deletion
     1. If height of children differ by more than 1
     2. Rotate children until subtrees are balanced
     3. Repeat check for parent (until root reached)

Tree Rotations

- **Changes shape of tree**
  - Rotation moves one node up in the tree and one node down
  - Height is decreased by moving larger sub-trees up and smaller sub-trees down
- **Types**
  - Single rotation
    - Left
    - Right
  - Double rotation
    - Left-right
    - Right-left

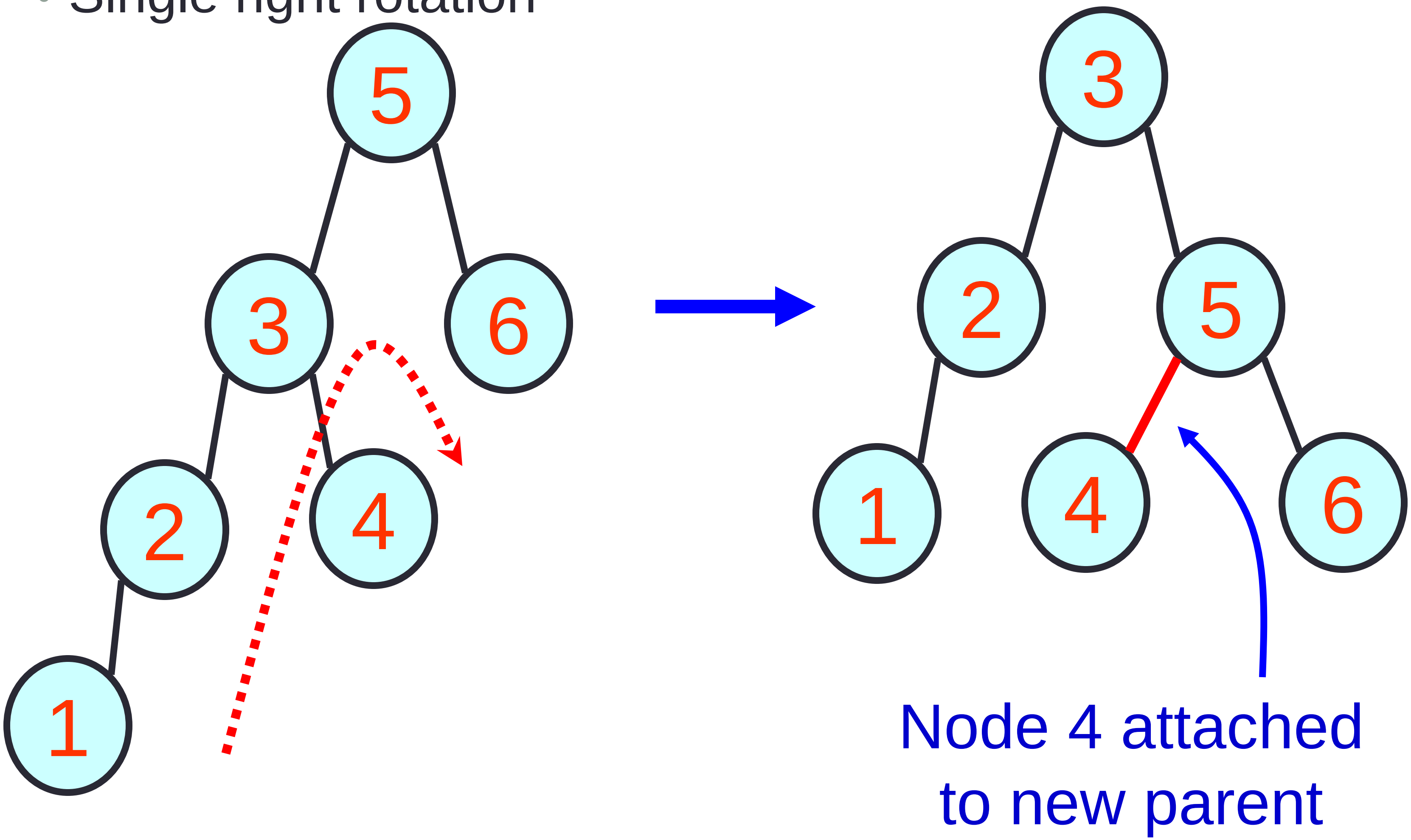
Tree Rotation Example

- Single right rotation



Tree Rotation Example

- Single right rotation

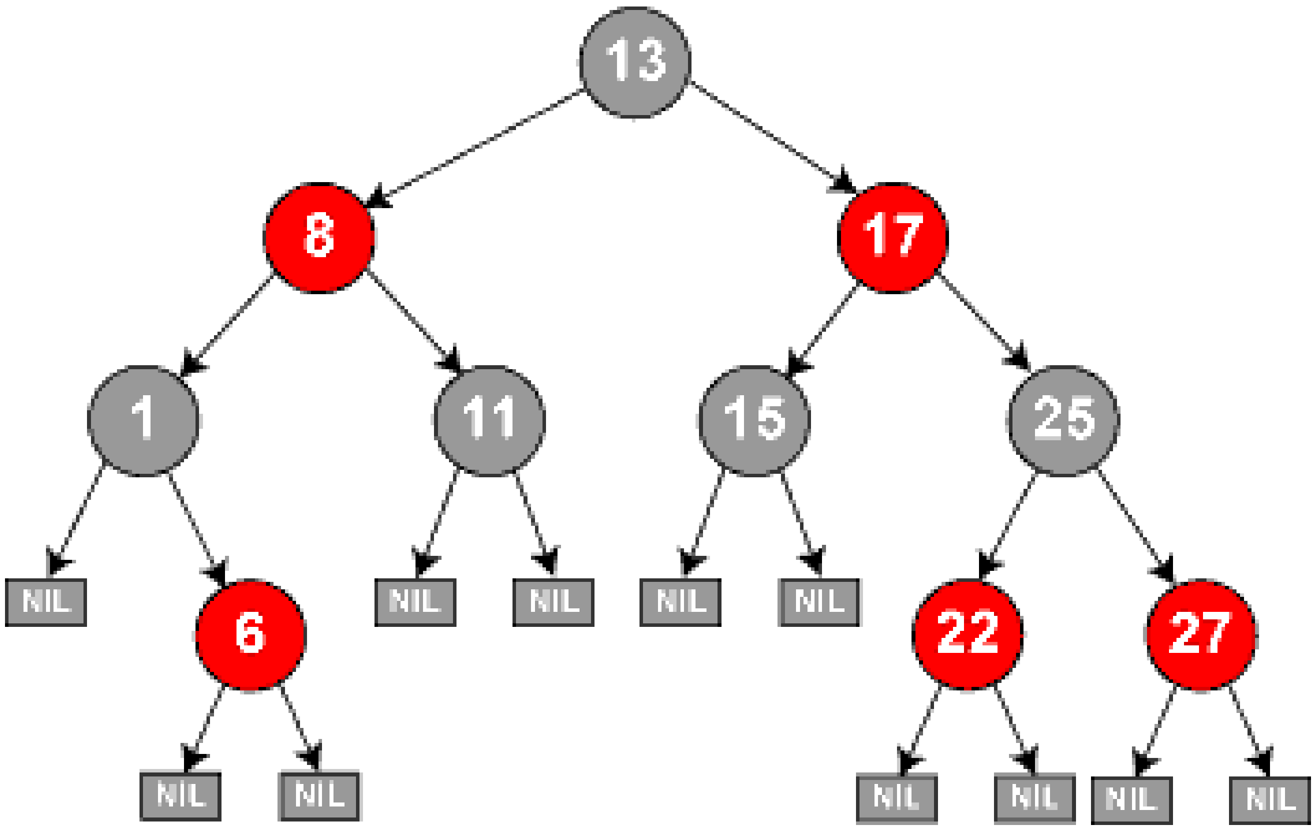


Red-black Trees

- **History**
  - Discovered in 1972 by Rudolf Bayer
- **Algorithm**
  - Insert / delete may require complicated bookkeeping & rotations
- **Java collections**
  - TreeMap, TreeSet use red-black trees
- **Properties**
  - Binary search tree
  - Every node is red or black
  - The root is black
  - Every leaf is black
  - All children of red nodes are black
  - For each leaf, same # of black nodes on path to root
- **Characteristics**
  - Properties ensures no leaf is twice as far from root as another leaf

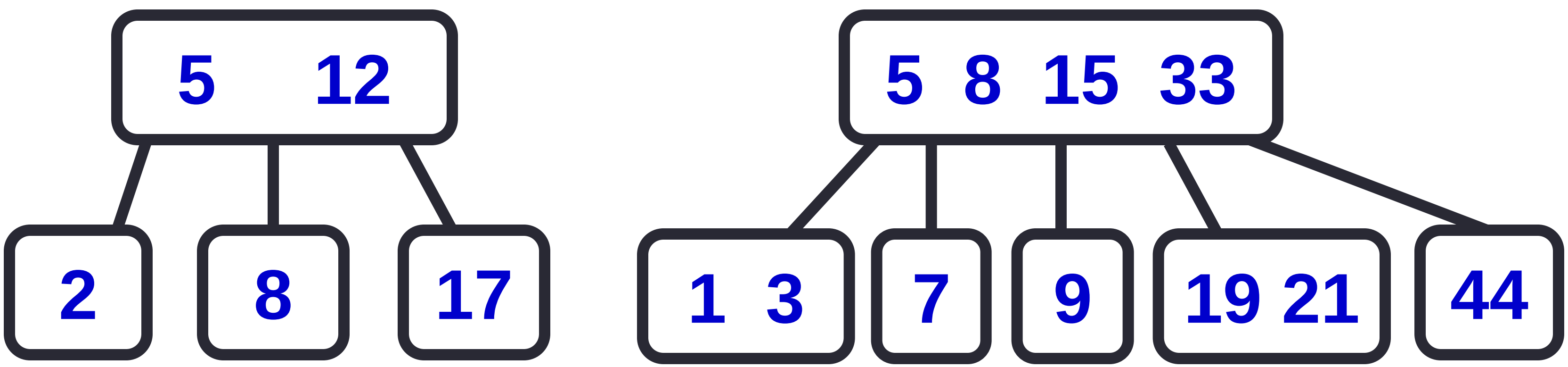
Red-black Trees

- Example



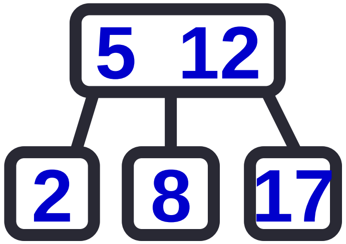
Multi-way Search Trees

- **Properties**
  - Generalization of binary search tree
  - Node contains 1…k keys (in sorted order)
  - Node contains 2…k+1 children
  - Keys in j^{th} child < j^{th} key < keys in $(j+1)^{th}$ child
- **Examples**

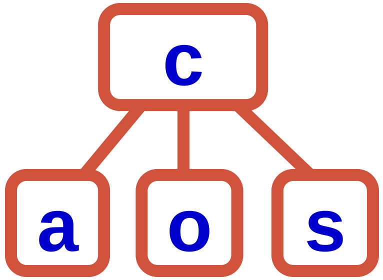


Types of Multi-way Search Trees

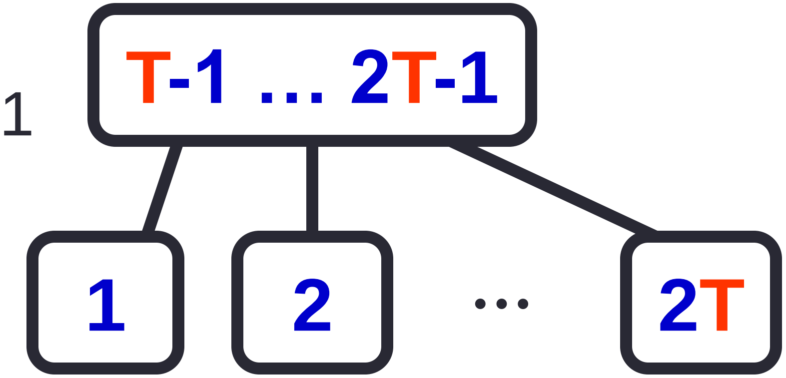
- **2-3 tree**
  - Internal nodes have 2 or 3 children



- **Index search trie**
  - Internal nodes have up to 26 children (for strings)

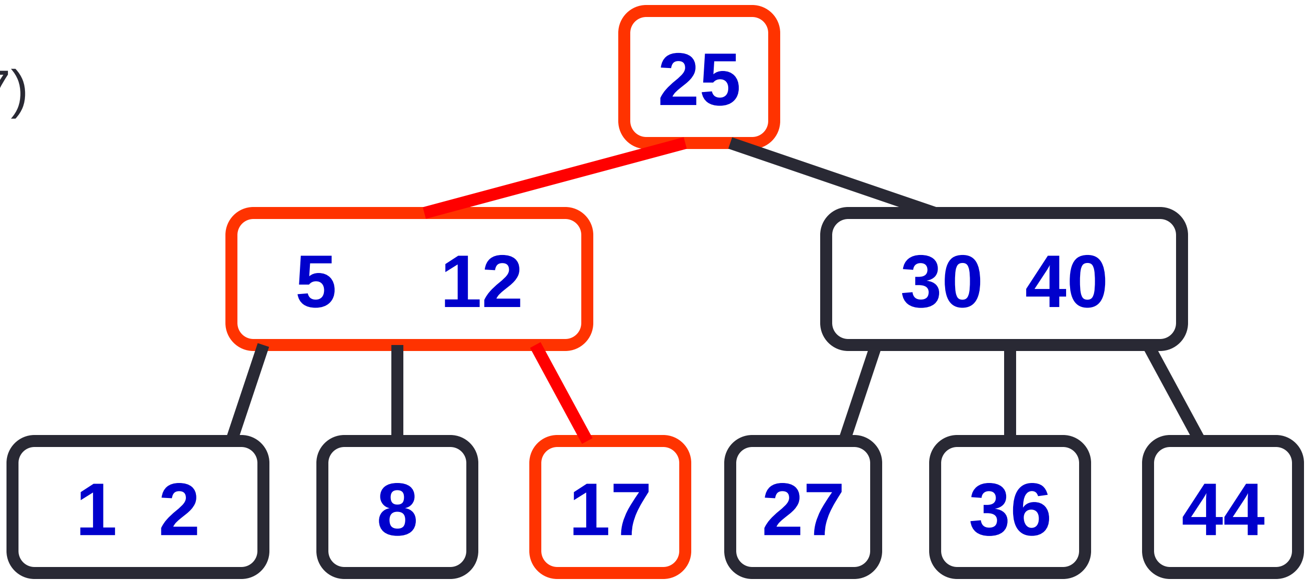


- **B-tree**
  - T = minimum degree
  - Non-root internal nodes have T-1 to 2T-1 children
  - All leaves have same depth



Multi-way Search Trees

- **Search algorithm**
    1. Compare key x to 1…k keys in node
    2. If x = some key then return node
    3. Else if (x < key j) search child j
    4. Else if (x > all keys) search child k+1
- **Example**
    - Search(17)

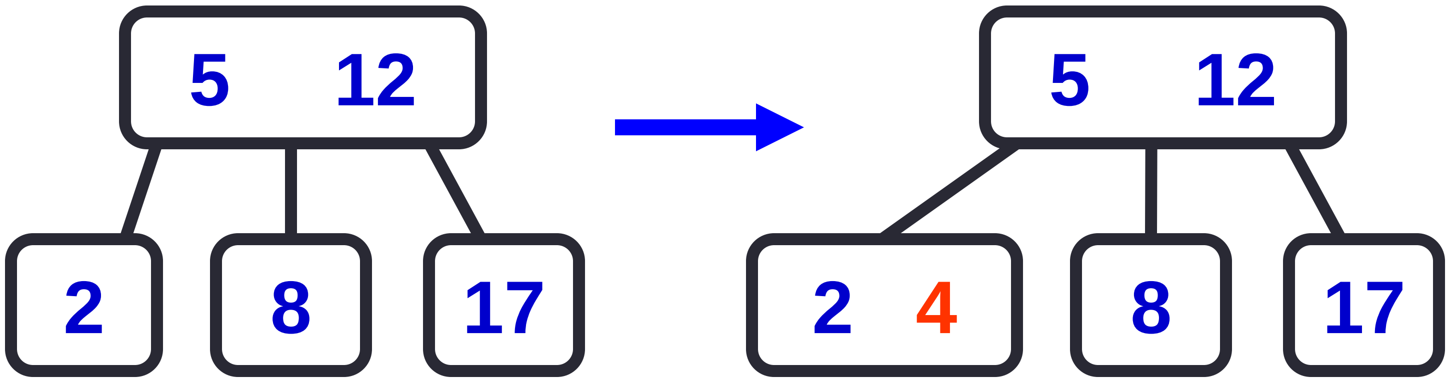


Multi-way Search Trees

- Insert algorithm
  1. Search key x to find node n
  2. If (n not full) insert x in n
  3. Else if (n is full)
     a) Split n into two nodes
     b) Move middle key from n to n's parent
     c) Insert x in n
     d) Recursively split n's parent(s) if necessary

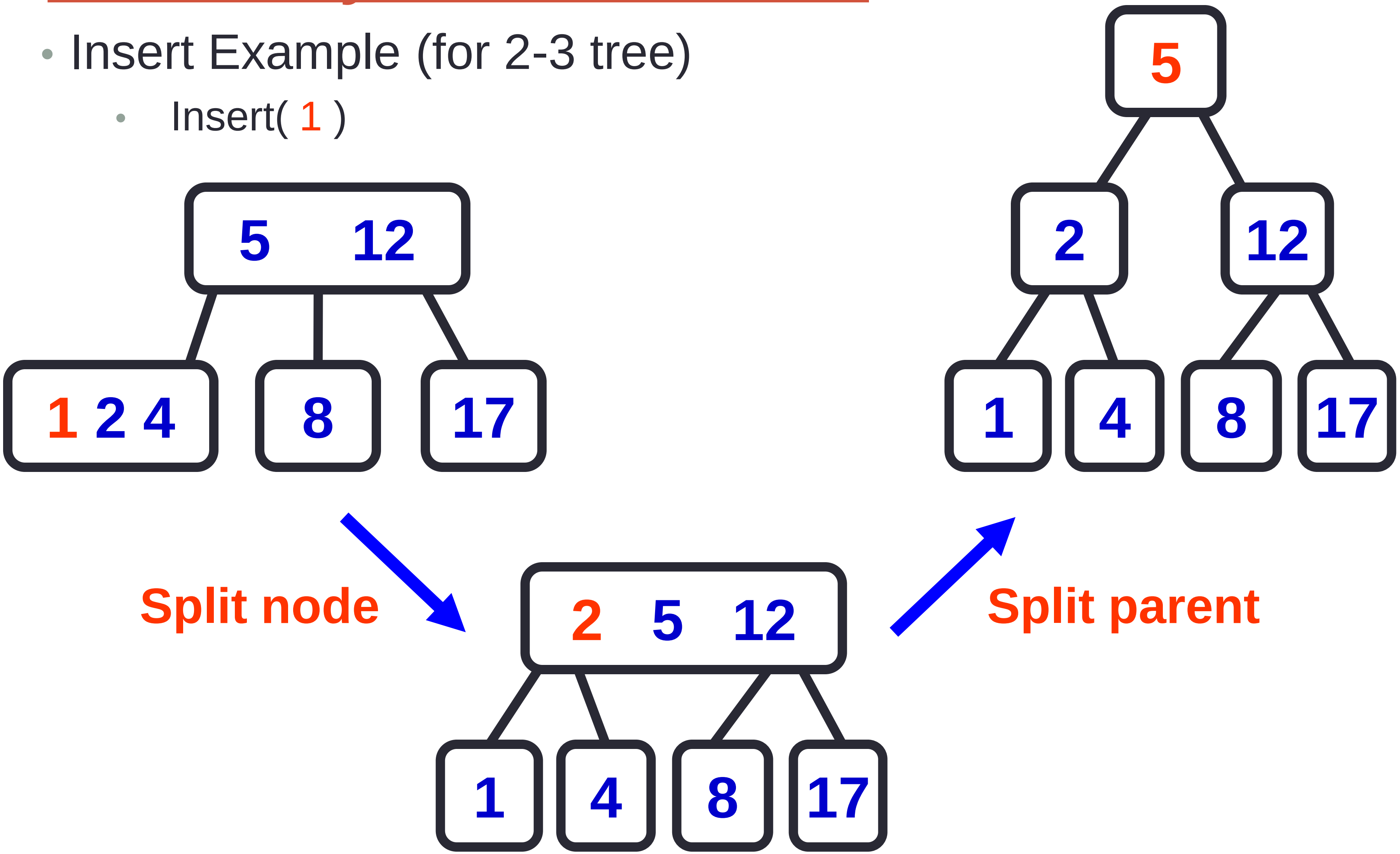
Multi-way Search Trees

- Insert Example (for 2-3 tree)
  - Insert(4)



Multi-way Search Trees

- Insert Example (for 2-3 tree)
  - Insert(1)

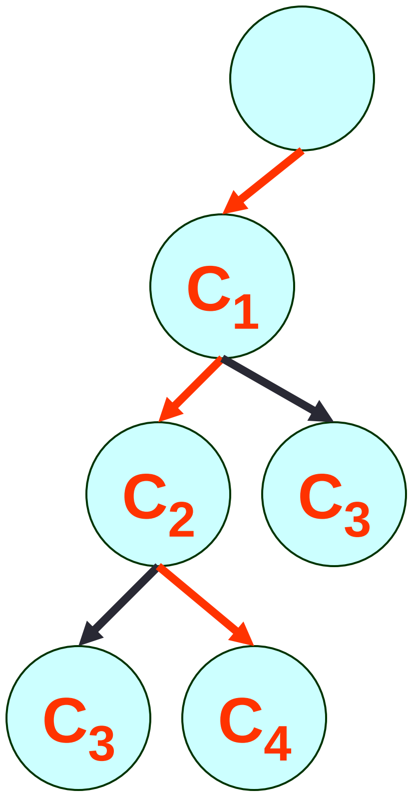


B-Trees

- **Characteristics**
  - Height of tree is O($\log_T(n)$)
  - Reduces number of nodes accessed
  - Wasted space for non-full nodes
- **Popular for large databases (indices)**
  - 1 node = 1 disk block
  - Reduces number of disk blocks read

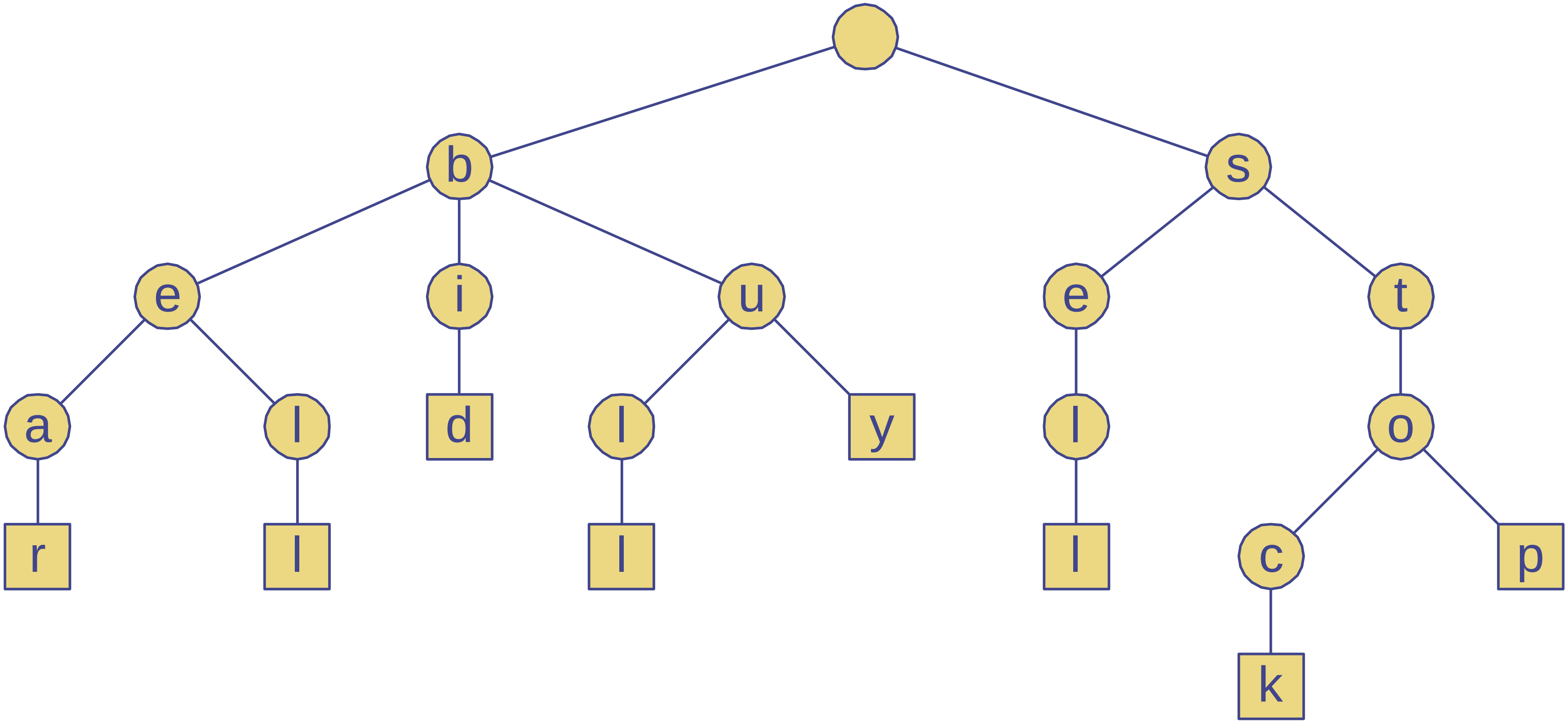
Indexed Search Tree (Trie)

- Special case of tree
- Applicable when
  - Key C can be decomposed into a sequence of subkeys C_1, C_2, … C_n
  - Redundancy exists between subkeys
- Approach
  - Store subkey at each node
  - Path through trie yields full key



Standard Trie Example

- For strings
  - { bear, bell, bid, bull, buy, sell, stock, stop }



Word Matching Trie

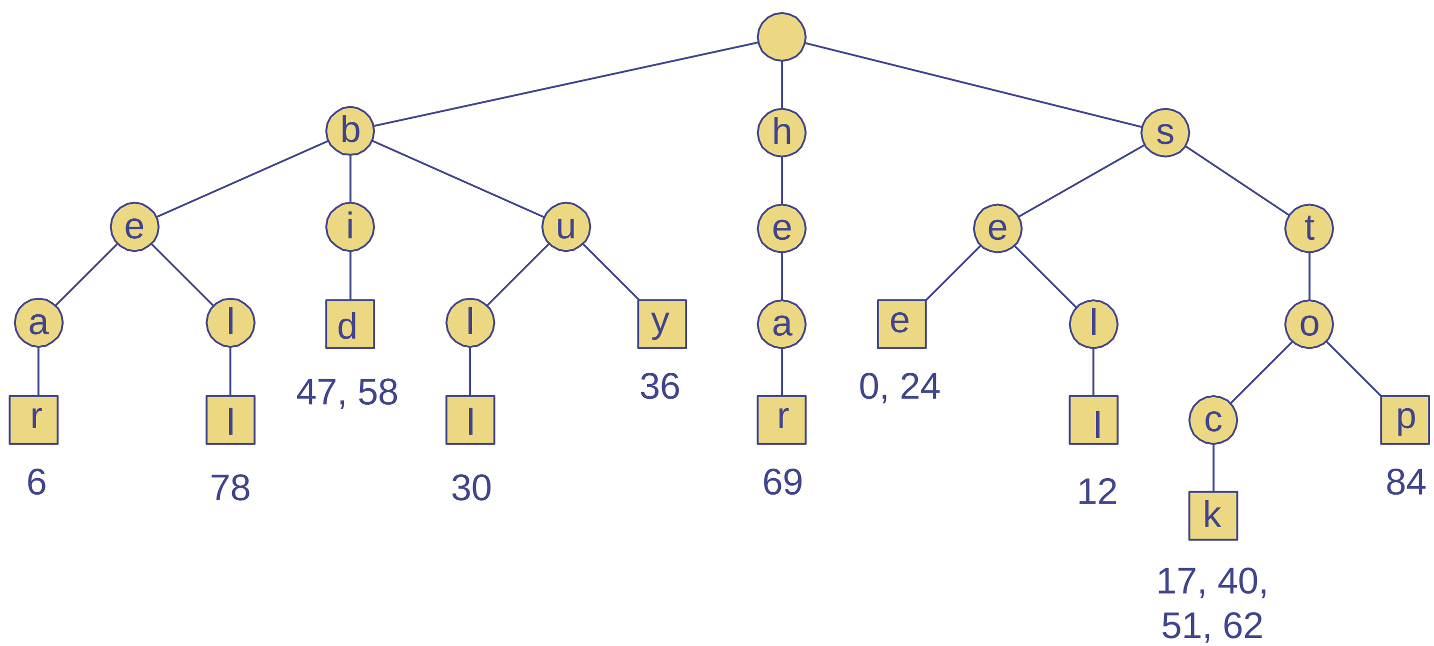
- Insert words into trie
- Each leaf stores occurrences of word in the text

s	e	e		a		b	e	a	r	?		s	e	l	l		s	t	o	c	k	!	
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23

s	e	e		a		b	u	l	l	?		b	u	y		s	t	o	c	k	!	
24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46

b	i	d		s	t	o	c	k	!		b	i	d		s	t	o	c	k	!	
47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68

h	e	a	r		t	h	e		b	e	l	l	?		s	t	o	p	!
69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88

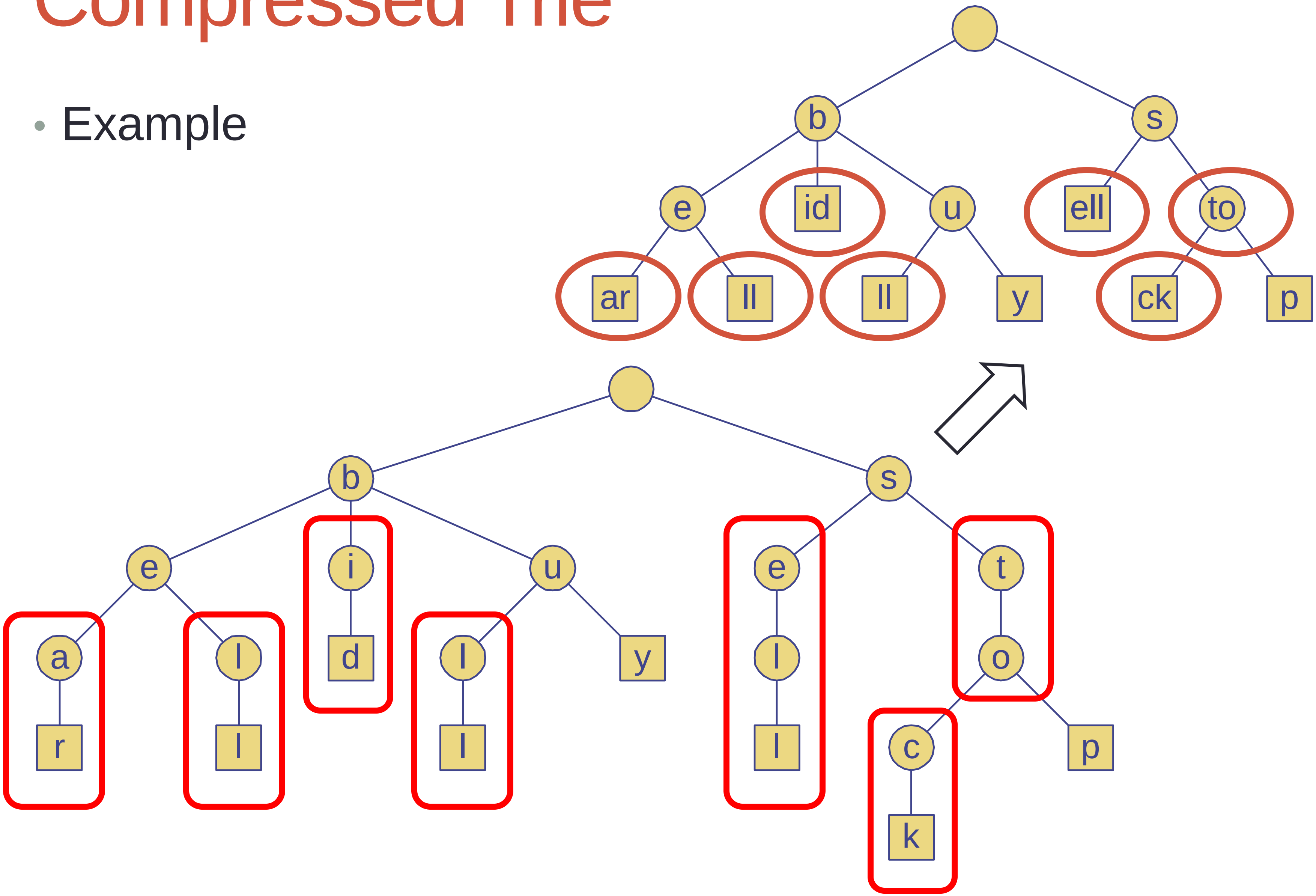


Compressed Trie

- **Observation**
  - Internal node v of T is redundant if v has one child and is not the root
- **Approach**
  - A chain of redundant nodes can be compressed
    - Replace chain with single node
    - Include concatenation of labels from chain
- **Result**
  - Internal nodes have at least 2 children
  - Some nodes have multiple characters

Compressed Trie

- Example



Tries and Web Search Engines

- **Search engine index**
  - Collection of all searchable words
  - Stored in compressed trie
- **Each leaf of trie**
  - Associated with a word
  - List of pages (URLs) containing that word
    - Called occurrence list
- **Trie is kept in memory (fast)**
- **Occurrence lists kept in external memory**
  - Ranked by relevance