

FOL

Language

- const symbols
- pred "
- quantifiers $\forall \exists$
- $\rightarrow, \neg, \wedge, \vee$

interpretations

each interp

corresponds to one Hbl row.

Sep 15-2:02 PM

Consider a FOL
with symbols
 P, a, b

Let I_1 be interp
where D_1 is the
people in the room,
at is K , and
 b^{I_1} is me, and P
will mean "is
at least 10"

Sep 15-2:12 PM

Consider the wff
 $P(a)$.

In I_1 , $P(a)$ means

K is at least 10.

$P(b)$, in I_1 , means

I am at least 10

Sep 15-2:18 PM

We say $P(a)$ and
 $P(b)$ are true in I_1 .
Also $\forall x P(x)$ is true
in I_1 , because
we're all ≥ 10 !

Sep 15-2:19 PM

Let I_2 have

$D_2 =$ a kindergarten
class

$= \{k_1, k_2, \dots, t\}$
 $\quad \quad \quad a \quad \quad \quad b$

and $P: \geq 10$ y.

$P(a)$: false in I_2

$P(b)$: true " "

$\forall x P(x)$: false in I_2

$\exists x P(x)$: true in I_2

Sep 15-2:24 PM

Language a, b
 $P()$
 $Q(,)$

I_3 :

$D_3 = D_1 =$ all $\neq \emptyset$

$P^{I_3} = P^{I_1} = " \geq 10 "$

$a^{I_3} = k, b^{I_3} = me$

$Q^{I_3}(x, y)$:

x older than y .

Sep 15-2:31 PM

$P(a)$ true in I_3 ✓
 $P(b)$ " " ✓
 $Q(b, a)$ true " ✓
 $\sim Q(a, b)$ " ✓

I_4 : Q means
 "at least
 as old as"

$\forall x \exists y Q(x, y)$ I_3, I_4
 F T
 (false $x=y$) ✓

Sep 15-2:35 PM

Number example
 Let I be:
 $D = \mathbb{N}$, nat. nums
 $= \{0, 1, 2, \dots\}$
 $=$ non-neg ints.
 $= \mathbb{Z}_{\geq 0}$
 and P : positive
 $S(,) : >$
 $U(x, y, z)$:
 $xy = z$
 $(\exists x P x)^I$: true
 $(\exists x \sim P x)^I$: true

Sep 15-2:45 PM

$[S : >, U : xy = z]^I$
 $(\forall x P x)^I$: false, bec
 $(\sim \forall x P x)^I$: true
 $\exists x \sim P x$ (prev page)
 are $\vee$ equivalent.
 logically
 (they come out the
 same in every
 interpretation
 (as each other))

Sep 15-2:52 PM

$S : >, U : xy = z, I : 1$
 $\forall x \forall y [(S(x, y) \wedge S(y, z))$
 $\rightarrow \sim U(x, y, z)]$
 This expresses (in I)
 that 2 is prime.

Sep 15-3:02 PM

$(\sim \forall x P x) \equiv (\exists x \sim P x)$
 $\iff$

recall
 $\sim (P \wedge Q) \equiv \sim P \vee \sim Q$

$\sim \forall x P x$
 $\iff$
 $\exists x \sim P x$

Sep 15-3:09 PM

Arguments

$\forall x P x$	} premises
$\cancel{\forall x P x}$	
$P a$	
$\forall x (P x \rightarrow Q x)$	
$P a$	
$Q a$	

Sep 15-3:12 PM