Fall 2011 **CMSC 451: Homework 5** Clyde Kruskal

Due at the start of class Tuesday, November 22, 2011.

Problem 1. Use the dynamic programming algorithm to find by hand an optimal parenthesization for multiplying matrices of dimensions are given by the sequence

$$< 6, 3, 10, 5, 8, 4, 20 > \quad .$$

Show the table. You may use a calculater.

Problem 2. Write an efficient algorithm to determine an order of evaluating the matrix product $M_1 \times M_2 \times M_3 \ldots \times M_n$ so as to minimize the scalar multiplications in the case where each M is of dimension $1 \times 1, 1 \times d, d \times 1, d \times d$ for some fixed d.

Problem 3. The number of combinations of n things taken m at a time $\binom{n}{m}$ can be computed using the following recurrence:

$$\binom{n}{m} = \binom{n-1}{m} + \binom{n-1}{m-1} \qquad \text{for } 0 < m < n$$

and

$$\binom{n}{0} = \binom{n}{n} = 1 \ .$$

(a) Write a recursive algorithm to compute $\binom{n}{m}$ using the above recurrence.

(b) Show that the worst-case running time of your algorithm can be exponential in n (depending on the value of m).

(c) Produce a *memoized* version of your algorithm.

(d) Give a dynamic programming algorithm to compute $\binom{n}{m}$.

(e) Analyze the running time of your dynamic programming algorithm as a function of m and n.

Problem 5. Do Exercise 6 on pages 317-318 of Kleinberg and Tardos.