Due Friday, December 7, 2012.

Problem 1. The *Hamiltonian path* problem (HP) is: Given an undirected graph $G = (V, E)$, does there exist a simple path that passes through every vertex of G? Show that the Hamiltonian path problem is in NP.

Problem 2. A *Matching* in an undirected graph $G = (V, E)$ is a set of edges (from the graph) such that no two have a vertex in common. A *Perfect Matching* is a matching of size $V/2$. A *grid graph of size* $m \times n$ is a graph where the vertices are placed in an $m \times n$ grid and each vertex is connected to its north, south, east, and west neighbors (if it has one).

(a) Prove that if either m or n is even, a grid graph of size $m \times n$ has a perfect matching.

(b) Prove that if both m and n are odd, a grid graph of size $m \times n$ does not have a perfect matching.

(c) A *Maximum Matching* is a matching with as many edges as possible. If both m and n are odd, what is the size of a maximum matching? Justify.

Problem 3. Given an undirected graph $G = (V, E)$, we would like to find a maximum matching. This is the "optimization" version of the Maximum Matching problem.

(a) What is the "decision" version of Maximum Matching?

(b) Show that (the decision version of) Maximum Matching is in NP.

(c) Show that if you could solve the optimization version of Maximum Matching in polynomial time that you could also solve the decision version in polynomial time.

(d) Show that if you could solve the decision version of Maximum Matching in polynomial time that you could also solve the optimization version in polynomial time. HINT: First find the size of the maximum matching, and then find the matching itself.