Fall 2012 **CMSC 351: Homework 5** Clyde Kruskal

Due by 2:30pm, Friday, October 12.

Problem 1. Assume that you guess that

$$\sum_{k=1}^{n} k^2 \;=\; an^3 + bn^2 + cn + d \;.$$

Use constructive induction to find the constants a, b, c, and d and prove that your guess was correct.

Problem 2. Do not use integrals for this problem. Do not worry about floors and ceilings (so you may assume that n is "nice"). Ignore second order terms. Consider

$$\sum_{k=1}^{n} k^2 \;.$$

(a) Split the sum into two equal-sized regions to obtain an upper bound for its value (as done in class).

(b) Split the sum into two equal-sized regions to obtain a lower bound for its value (as done in class).

(c) Show how to obtain a better upper bound by splitting the sum into two unequal-sized regions. Make your bound as tight as possible. How does your bound compare with the upper bound obtained in Part (a)?

(c) Show how to obtain a better lower bound by splitting the sum into two unequal-sized regions. Make your bound as tight as possible. How does your bound compare with the lower bound obtained in Part (b)?

Problem 3.

(a) Use the integral method to obtain upper and lower bound bounds for

$$\sum_{k=1}^{n} k^2 \;.$$

(b) How do your bounds compare with those obtained in Problem 2?

(c) How do your bounds compare with the exact polynomial in Problem 1?

Problem 4. Use the ratio method to find reasonably tight upper and lower bounds for

$$\sum_{k=1}^{\infty} \frac{k^3}{2^k} \;.$$

Note that very small values of k will not provide a ratio less than 1. You may use a calculator, but justify your answer.