2	3	4	5	6
3	4	5	6	8
4	5	$\boxed{6}$	8	5
5	7	8	9	3
9	10	9	4	3

Figure 1: Image patch

(1) (10 pts) (a) For the image patch in Figure 1 at the pixel at the center (that is the pixel marked by the black square) apply the following filters and round to the nearest integer value:

i. a 3×3 Gaussian filter $\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

ii. a 3×3 Box filter (that is averaging in a 3×3 neighborhood).

(b) Compute the edge direction and strength (that is the direction and absolute value of the image gradient) at the center pixel using the masks of the Sobel edge detector.

$$S_1 = \frac{1}{8}\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad S_2 = \frac{1}{8}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

(c) Apply a median filter to the center pixel. Explain for what kind of noise median filtering will work best.

(d) Why is the Gaussian filter a good smoothing filter? (How can it be implemented fast? How can we implement repeated Gaussian filtering in one operation?)

(e) What happens to the two edges at the boundaries of a dark line on a white background if the image is smoothed with a Gaussian with kernel size larger than the width of the line?

(f) Explain why Box filtering (that is averaging) attenuates the noise.

(g) Explain the concept of aliasing and give an example.

(2) (10 pts) Consider the cube with points $P1, P2, P3, P4, P5, P6, P7, P8$ and 3D coordinates in the world coordinate system as given in Figure 2. A

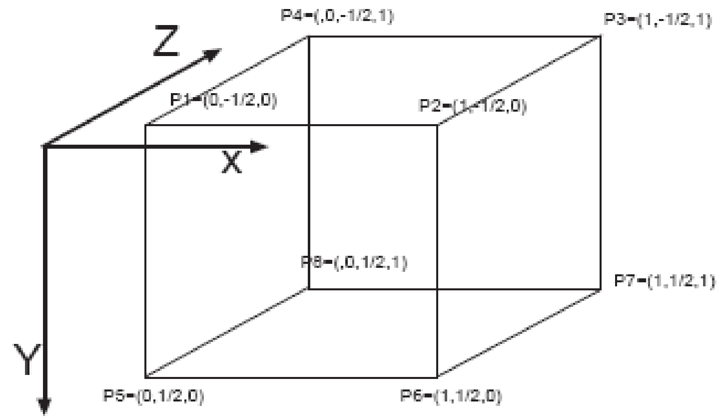

Figure 2:

calibrated camera with focal length $f = 1$ whose origin is at $(0, 0, -3)$ and which has a rotation of $-45°$ around the Y-axis with respect to the world coordinate system takes an image of the cube. The image ccoordinates of the corners of the cube are labeled $p1, p2, p3, p4, p5, p6, p7, p8$ Remember a rotation of angle α around the Y-axis can be expressed by the rotation matrix $R = \begin{pmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{pmatrix}$, and $\cos(-45°) = \frac{1}{2}\sqrt{2}$, and $\sin(-45°) = -\frac{1}{2}\sqrt{2}$.

(a) Derive the projection matrix mapping homogeneous world coordinates to homogeneous image coordinates.

(b) Compute the homogeneous and the non-homogenous image coordinates of points $p5, p6, p7, p8$.

(c) Derive the non-homogenous coordinates of the 3 vanishing points, corresponding to the 3 parallel lines.

(d) Compute the vanishing point of the line $P\bar{5}P7$

(e) How would the camera need to be positioned with respect to the cube, such that 2 of the vanishing points are ideal (that is are at infinity)?

(3) (5pts) Describe the Canny edge detector. Exlain its three modules.

(4) (10pts) Show that an affine transformation can map a circle to an ellipse, but cannot map an ellipse to a hyperbola.

(5) (20 pts) Show that there is a three parameter family of projective transformations which fix a unit circle at the origin (i.e. they map a unit circle at the origin to a unit circle at the origin). What is the geometric interpretation of this family?

(6) (20pts) Suppose we are given a 3X4 camera matrix:

3.53e+2	3.39e+2	2.77e+2	-1.44e+6
-1.03e+2	2.33e+1	4.59e+2	-6.32e+5
7.07e-1	-3.53e-1	6.12e-1	-9.18e+2

Compute the camera center and the calibration parameters

(7) (20 pts) Consider 3 points a,b,c lying on a straight line in an image. They are images of points A,B and C lying on a line in 3D. Show how to compute the vanishing point of line ABC from the ratio: ab/bc that can be measured in the image.

(8) (5pts) In a 3X4 camera matrix, what is the geometric meaning of the 4 column vectors?