Paradoxes of Human Decision Making

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We sometimes make decisions that are inconsistent with formal models of decision making. The phenomenon, in my opinion, is interesting because it shows us that common sense reasoning of human decision making is not necessarily an instance of a formal model, such as logics. In this report, we will look at two well-known paradoxes of human decision making, the Ellsberg Paradox and Conjunction Fallacy.

1. Ellsberg Paradox

Before we discuss the Ellsberg Paradox, it is helpful to mention the Expected Utility Theory, of which the paradox is a violation and Risk Aversion, which is a part of the Expected Utility Theory, and an explanation of our preference of certainty to risk.

In economics, utility expresses a personal preference of a person with regard to a particular outcome; it is correlated to the person's Desire or Want of that outcome. The Expected Utility Theory (EUT) models a personal preference as a function of the utility, probabilities of the outcomes, and risk aversion (the attitude toward risk); and it explains an individual's choices (bets) with regard to uncertain outcomes, such as gambles. EUT has been used widely to explain human decision making in areas such as gambling and insurance.

Uncertain outcomes involve risks, and risks have tolerance. A person prefers buying bonds to buying stocks because bonds are considerably less risky than stocks. Risk Aversion describes just that: it is the "reluctance to accept a bargain with an uncertain payoff rather than another bargain with a more certain, but possibly lower expected payoff"[1].

Ellsberg Paradox is a conclusion of the following 1-urn experiment:

> *An urn contains 30 red balls, and 60 of either yellow or black balls (the ratio of black and yellow balls is unknown – a Knightian uncertainty). At every trial, a person picks a ball from the urn. In the first game, (S)he is also given a choice between 2 gambles that he can play:*
>
> > *Gamble A: If the ball is red, (s)he receives $100*
> > *Gamble B: If the ball is black, (s)he receives $100*
>
> *In a different game, (s)he is given another choice between 2 different 2 gambles:*
>
> > *Gamble C: If the ball is red or yellow, (s)he receives $100*
> > *Gamble D: If the ball is black or yellow, (s)he receives $100*

A mathematical demonstration of Expected Utility Theory goes like this:

> Let us call R, B, and Y the *estimated* probabilities of the chosen ball is Red, Black and Yellow respectively. We know that R is 1/3, and B + Y is 2/3, but for the mathematical analysis, this does not make any difference. Let U(.) be the utility of the payoff, and since we strictly prefer $100 to $0, it follows that U($100) > U($0). Now, let us suppose that in the first game, the person choose Gamble A,
>
> > R * U($100) + (1-R) * U($0) > B * U($100) + (1-B) * U($0)

⬄ R * [U($100) - U($0)] > B * [U($100) - U($0)]

⬄ R > B

> That is, a choice of Gamble A implies the supposition that there are more Red balls than Black balls. Similar result is given for the second game: if the person choose Gamble C, (s)he thinks that there should be more Red balls than Black balls. Consequently, a selection of Gamble A over Gamble B in the first game implies a selection of Gamble C over Gamble D in the second game.

However, when surveyed, most people strictly prefers gamble A to B, and D to C, which is a violation of the Expected Utility Theory [2]. This is Ellsberg Paradox.

There are various attempt to provide decision-theoretic explanation for the Ellsberg Paradox [2]. One such attempt is based on the info-gap decision theory. Intuitively, since we don't know the precise probabilities of B and Y, instead of evaluate the precise expected utility, info-gap tries to maximize the robustness against severe uncertainty of the outcomes. The robustness is the changes (or sensitivity) of the expected utility with respect to the perturbation of parameters in the model. It turns out that, when we look at the paradox from the perspective of info-gap theory, the choices are consistent.

Another psychological explanation is that the betting games triggers our deceit aversion mechanism (i.e. the untold probabilities of the outcome are there to deceive). Therefore, it is better to choose the Gambles whose probabilities of loosing are better known: the probability of loosing Gamble A is 2/3, and of loosing Gamble B is unknown; the probability of loosing Gamble C is 1 unknown, and of loosing gamble D is 1/3. However, the psychological explanation is based on the fact that the person is unaware that the urn is unchanged from the first game to the second game (because if this is the case, a selection of Gamble A (there are more Red balls than Black balls) should be logically followed by a selection of Gamble C as in the mathematical demonstration).

2. Conjunction Fallacy

Amos Tversky and Daniel Kahneman construct an experiment with results as following:

> *Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.*
>
> *Which is more probable?*
>
> *(1) Linda is a bank teller (A).*
>
> *(2) Linda is a bank teller (A) and is active in the feminist movement (B).*
>
> *When asked, 90% of the people choose (2).*

Clearly, the event in (2) is contained within the event in (1); therefore it should be the opposite that (1) is more probable. An alternative explanation is that P(2) = P(A, B) = P(A) P(B|A) ≤ P(A) = P(1), where P(x) is the probability of the event x. This is an instance of conjunction fallacy, where a specific event is more probable than the general event that contains it.

Tversky and Kahneman explain that most people choose (2) because it is more representative (more specific) of Linda than (1) – a form of representative heuristic, but as we have seen in the probability analysis above, because it is representative, it does not mean that it is more probable. In this sense, it is an instance of extension neglect, which is a cognitive bias that the “size of the set has little or no influence on its valuation,” or the probability of an event is less important.

A critic of the Linda’s experiment is from Gerd Gigerenzer and Ralph Hertwig where they claim that the wording of the experiment may allow violations of the conversational relevancy maxim [4]. They argue that some of the terms in the experiment are semantically ambiguous. For instance, the intended meaning of “probable” is mathematical probability, or what happens frequently, when what people generally understand that “probable” means “what is more plausible.” Another example is the word “and” whose intended meaning is “intersection” in the probability sense, but is “union” in the general sense of the people. Gigerenzer also shows that if the Linda’s experiment is changed to disambiguate these polysemous terms, none of the participants give the wrong answer [4].

3. Conclusion

It is interesting to see that our decisions sometimes are more than just instances of formal reasoning. Psychological effects, such as representative heuristics, can also play important roles. To have a theory that can explain our behavior in making our decisions is a difficult endeavor and practical experiments like the Ellsberg’s and Linda’s are very helpful to our understanding of the theory.

References

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[5] http://en.wikipedia.org/wiki/Extension_neglect
[6] Mellers, Barbara; Ralph Hertwig & Daniel Kahneman (2001). "Do frequency representations eliminate conjunction effects? An exercise in adversarial collaboration". Psychological Science 12 (4): 269–275.