Sorites Paradox Report

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1. The Paradox

The sorites paradox is a form of inconsistency created by the vagueness of certain predicates in language. It is first attributed to the logician Eubulides of Miletus, who presented it in the following way: If you have one grain of wheat lying on the floor, would you say you have a heap of wheat? No. What about two grains? Clearly, still no. However, it is also clear that for some number of grains lying on the floor you would say yes, that is a heap of wheat. Where then do you draw the line? More formally, for what positive integer n is it the case that n grains of wheat is not a heap, but n + 1 grains is?

Intuitively, it is hard to imagine a situation where one additional grain could make the difference between a heap and a non-heap. Similar paradoxes can be constructed in a wide variety of situations. Examples include baldness (a man with one or two hairs on his head is considered bald and a man with 10,000 hairs is not, but how many hairs are enough to not be bald?), crowd-ness (1000 people are a crowd, and, old sayings notwithstanding, 4 people are not, but what is the minimum?), and many others in a similar vein. What these examples share is that the predicate involved is both vague and in some sense quantifiable. There is no paradox of this sort involving the predicates “5-fingered” or “underground” (an object might be partly underground, but there would be no lack of clarity about the underground-ness of any part of that object). It is also hard to conceive of a sorites paradox involving courage or beauty, since these predicates cannot easily be defined in terms of some discreet quantity and anything.

Now, with an outline of what a sorites paradox is, the next question is why should we care? Some readers of a mathematical bent may have noticed one consequence. The formulation above is a puzzle that suggests something strange is going on. However, with a small adjustment, we can change it into a proof:

(1) One grain of wheat is not a heap.
(2) Adding one additional grain will never be enough to go from a non-heap to a heap.
(3) From 1 and 2, by induction, no number of grains will ever constitute a heap.

Using this technique, we can prove that there are no heaps, that everyone is bald, and that the people attending a baseball game never amount to more than a small gathering. Additionally, for every proof of the form above, there exists a negative version:

(1) 100,000 grains of wheat are a heap
(2) Taking away a single grain of wheat from a heap will never reduce it to a non-heap
(3) Therefore, by induction, any number of grains is a heap.

Clearly, there is something strange and problematic going on here. Specifically, it would appear that one of the following is true:

(1) One of our premises is false
(2) The logic is invalid
(3) Standard logical formulations are insufficient to deal with the vagueness pointed out by sorites paradoxes

The rest of this report will be concerned with examining various efforts to defend each of these three responses.

2. Responses

The most basic approach to (2) above, that some mistake has been made in the logical argument, does not seem particularly persuasive – the arguments seem clearly valid based on the logical systems we are used to. However, a more interesting case can be made through a combination of (2) and (3) – by asserting that we need additional logical structures to deal with sorites-style predicates, which can fit in a framework similar to the logic we are used to. One such approach is a non-bivalent supervaluation logic adopted by Dummett (1975), Fine (1975) and Keefe (2000). In this formulation, it is possible for a statement to be wholly true, wholly false, or a third value which is conditionally true or false depending on how the relevant semantic indecision is resolved.

This approach does seem to defeat the inductive proofs above, but it does so in a strange way. Specifically, it is not true that adding a single grain will never produce a heap. There is no instance for which this is false, but there are instances for which it is not wholly true, and that is sufficient to render the statement of universal truth itself false. Even more strangely, this approach commits supervaluationists to both of the following statements:

a. True $(\exists n\ (Fa_n \ \&\ {\sim}Fa_{n+1}))$
b. ${\sim}\exists n\ (\mathrm{True}(Fa_n \ \&\ {\sim}Fa_{n+1}))$

In other words, it is true that there is *some* point at which one additional grain turns a non-heap into a heap, but for any *specific* n it is not true that it is such a point. This is a very subtle distinction, and one might argue that supervaluationists have eliminated one counterintuitive conclusion at the cost of introducing a worse one.

There are several related approaches, including the idea of many- and infinite-valued logics, where truth has levels and statements can be more or less true than each other with arbitrary precision. These also provide solutions of a form to the sorites paradox, but they raise several additional questions relating to the meaning and treatment of relative truth. Addressing these issues in detail is beyond the scope of this paper, but the reader may see Tye (1994) and Hyde(2008), respectively, for discussions of 3- and infinite-valued logics.

A different approach entirely was proposed by Kamp (1981) and has been extended by Stanley (2003) and others. Their idea is that the reason these terms are vague is that our idea of a heap, or our idea of baldness, is dependent not only on the number of grains or hairs but also on the context in which we observe them. When we see a pile of grain and say it is not a heap, in doing so we put ourselves in a context in which a pile one grain larger will not be a heap either. If 100 grains are added one by one, and we are asked after each one to state if we have a heap, we will be inclined to say no, since it is ridiculous to think that one additional grain could make the difference, and we have already established that the previous pile was no heap. However, if instead 100 grains are added at once, we might say that an identical grouping of grains, seen now in a different context, is in fact a heap. Stanley puts this idea as follows: "when we look for [a] boundary…our very looking has the effect of changing the interpretation of the vague expression so that the boundary is not where we are looking."

The contextual basis for sorites predicates seems to offer a fairly reasonable explanation for the paradox, though there are still some issues (see Stanley). Intuitively, it does seem that a definition of "heap" or "baldness" that relies solely on a count of grains or hairs is incomplete. It is also consistent with human psychology that people, having staked out a position that some object is or is not a heap, would discredit that position by saying that a nearly indistinguishable object should be classified differently. Finally, it allows us to deny a premise of the sorites argument: it is not true that a pile with one grain more than a non-heap cannot be a heap – it depends on context.

References:

1) Dummett, M., 1975. 'Wang's paradox', *Synthese* 30: 301–24; reprinted in his *Truth and Other Enigmas*, Also reprinted in Keefe and Smith (1996).
2) Fine, K., 1975. 'Vagueness, truth and logic', *Synthese*, 30: 265–300. Reprinted in Keefe and Smith (1996).
3) Hyde, D., 2008. *Vagueness, Logic and Ontology*, Aldershot: Ashgate.
4) Kamp, H., 1981. 'The paradox of the heap', in U. Mönnich (ed.), *Aspects of Philosophical Logic*, Dordrecht. Reidel.
5) Keefe, R., 2000. *Theories of Vagueness*, Cambridge: Cambridge University Press.
6) Stanley, J., 2003. 'Context, interest-relativity, and the sorites', *Analysis*, 63: 269–80.
7) Tye, M., 1994. 'Sorites paradoxes and the semantics of vagueness', in J. Tomberlin (ed.), *Philosophical Perspectives: Logic and Language*, Atascadero, California: Ridgeview. Reprinted in Keefe and Smith (1996).