

Name & Section Number: _____

For the following questions, do not worry about floors and ceilings (so you may assume that n is “nice”). Consider

$$\sum_{k=1}^n \frac{1}{k^2} .$$

1. Split the sum into two equal-sized regions to obtain an upper bound for $\sum_{k=1}^n \frac{1}{k^2}$.

2. Split the sum into two equal-sized regions to obtain a lower bound for $\sum_{k=1}^n \frac{1}{k^2}$.

3. Recall that in class (and in the book), we were able to show that (approximately) $\sum_{k=1}^n \frac{1}{k} \leq \lg(n+1)$ by splitting the sum into many (unequal-sized) regions. Use the same method to find an upper bound for $\sum_{k=1}^n \frac{1}{k^2}$.

4. Use the integral method to obtain upper and lower bound bounds for $\sum_{k=1}^n \frac{1}{k^2}$.