Fall 2013 **CMSC 351: Homework 9** Clyde Kruskal and Vibha Sazawal

Due at the start of class Wednesday, December 4, 2013.

Problem 1. Let $G = (V, E)$ be a directed graph.

(a) Assuming that G is represented by an adjacency matrix $A[1..n, 1..n]$, give a $\Theta(n^2)$-time algorithm to compute the adjacency list representation of G. (Represent the addition of an element v to a list l using pseudocode by $l \leftarrow l \cup \{v\}$.)

(b) Assuming that G is represented by an adjacency list $\mathrm{Adj}[1..n]$, give a $\Theta(n^2)$-time algorithm to compute the adjacency matrix of G.

Problem 2. Assume that a list of vertices represents a directed (not necessarily simple) cycle where the last vertex of the array has an edge to the first vertex. Assume you have two cycles in a (directed) graph $G = (V, E)$, represented by two arrays A and B, where the two cyces do not share any edges but do intersect (at at least one node). Describe an efficient algorithm to splice the two cycles together into one cycle represented by an array of nodes C. Give the pseudo code *and* briefly state in English how you algorithm works. (There may be more than one way to splice the two cycles together. Any legitimate splicing is fine.) Analyze its efficiency.

Problem 3. Assume you use breadth-first search to actually find your way out of a maze (consisting of rooms and hallways). One room is the start room, and one room is the exit room. We want to count how many hallways you need to walk down. If you walk down the same hallway twice that counts as two. Each direction counts once, so back-and-forth across a hallway counts as two. The first time you visit a room must be in the order of breadth-first search. Other than that you can (and must) be clever.

(a) Assume the maze is one long path of n rooms where you start on one end and the exit is at the other end. How many hallways do you walk down? Justify.

(b) Assume the maze consists of a start room with $n - 1$ other rooms directly connected to it. (In other words it is a star graph.) Assume the last room you visit is the exit room. How many hallways do you walk down? Justify.

(c) Assume the maze is a complete binary tree with $n = 2^k - 1$ rooms. Assume that the root is the start room. To keep things simple, assume the root is also the exit room but only *after* you have visited all of the other rooms. How many hallways do you walk down? Justify.