

COMMONLY USED LAWS, PROPERTIES & DEFINITIONS: LOGIC & SETS

CMSC 250

LOGIC

Given any statement variables p , q , and r , a tautology t and a contradiction c , the following logical equivalences hold:		
1. Commutative laws:	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. Associative laws:	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive laws:	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity laws:	$p \wedge t \equiv p$	$p \vee c \equiv p$
5. Negation laws:	$p \vee \neg p \equiv t$	$p \wedge \neg p \equiv c$
6. Double Negative law:	$\neg(\neg p) \equiv p$	
7. Idempotent laws:	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. DeMorgan's laws:	$\neg(p \wedge q) \equiv \neg p \vee \neg q$	$\neg(p \vee q) \equiv \neg p \wedge \neg q$
9. Universal bounds laws:	$p \vee t \equiv t$	$p \wedge c \equiv c$
10. Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. Negations of t and c:	$\neg t \equiv c$	$\neg c \equiv t$

Modus Ponens $p \rightarrow q$ p Therefore q	Modus Tollens $p \rightarrow q$ $\neg q$ Therefore $\neg p$		Disjunctive Syllogism $p \vee q$ $\neg q$ Therefore p	$p \vee q$ $\neg p$ Therefore q
Conjunctive Addition p q Therefore $p \wedge q$			Hypothetical Syllogism $p \rightarrow q$ $q \rightarrow r$ Therefore $p \rightarrow r$	
Disjunctive Addition p Therefore $p \vee q$	q Therefore $p \vee q$		Dilemma: Proof by Division into Cases $p \vee q$ $p \rightarrow r$ $q \rightarrow r$ Therefore r	
Conjunctive Simplification $p \wedge q$ Therefore p	$p \wedge q$ Therefore q		Rule of Contradiction $\neg p \rightarrow c$ Therefore p	
Closing C.W. without contradiction	$ p$ Assumed $ q$ derived Therefore $p \rightarrow q$		Closing C.W. with contradiction	$ p$ Assumed $ x \wedge \neg x$ derived Therefore $\neg p$

SETS, DEFINITIONS, ETC.

Definition of Implication	$p \rightarrow q$ $\neg(p \rightarrow q)$	$\equiv \neg p \vee q$ $\equiv p \wedge \neg q$
Definition of Biconditional	$A \leftrightarrow B$ $\neg(A \leftrightarrow B)$	$\equiv (A \rightarrow B) \wedge (B \rightarrow A)$ $\equiv (A \wedge \neg B) \vee (B \wedge \neg A)$
Negation of Quantifiers	$\neg \forall x P(x)$ $\neg \exists x P(x)$	$\equiv \exists x \neg P(x)$ $\equiv \forall x \neg P(x)$
Universal Modus Ponens	$\forall x \in D, P(x) \rightarrow Q(x)$ $P(a)$	$\rightarrow Q(a)$
Universal Modus Tollens	$\forall x \in D, P(x) \rightarrow Q(x)$ $\neg Q(a)$	$\rightarrow \neg P(a)$
Universal Instantiation	$\forall x \in D, P(x)$	$\rightarrow P(a)$
Existential Generalization	$P(a)$ where $a \in D$	$\rightarrow \exists x \in D, P(x)$
Universal Generalization**	$P(a)$ where $a \in D$	$\rightarrow \forall x \in D, P(x)$
Existential Instantiation **	$\exists x \in D, P(x)$	$\rightarrow P(a)$ where $a \in D$

** NOTE: Remember the special circumstances required for the rules marked by the stars.

Given any sets A , B , and C :	
1. Inclusion for Intersection:	$(A \cap B) \subseteq A$ $(A \cap B) \subseteq B$
2. Inclusion for Union:	$A \subseteq (A \cup B)$ $B \subseteq (A \cup B)$
3. Transitive Property of Subsets:	$(A \subseteq B) \wedge (B \subseteq C) \rightarrow A \subseteq C$

Given any sets A , B , and C , the universal set U and the empty set $\emptyset$:	
1. Commutative laws:	$A \cap B = B \cap A$ $A \cup B = B \cup A$
2. Associative laws:	$(A \cap B) \cap C = A \cap (B \cap C)$ $(A \cup B) \cup C = A \cup (B \cup C)$
3. Distributive laws:	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
4. Intersection with U (Identity):	$A \cap U = A$
5. Double Complement law:	$(A')' = A$
6. Idempotent laws:	$A \cap A = A$ $A \cup A = A$
7. De Morgan's laws:	$(A \cup B)' = A' \cap B'$ $(A \cap B)' = A' \cup B'$
8. Union with U (Universals Bounds):	$A \cup U = U$
9. Absorption laws:	$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$
10. Alternative Representation for Set Diff:	$A - B = A \cap B'$

Given any sets A, B :	
1. $A \subseteq B \rightarrow (A \cap B = A)$	Intersection with Subset
2. $A \subseteq B \rightarrow (A \cup B = B)$	Union with Subset

Given any sets A, B , and C , the universal set U and the empty set $\emptyset$:	
1. Union with $\emptyset$:	$A \cup \emptyset = A$
2. Intersection and Union with Complement	$A \cap A' = \emptyset$ $A \cup A' = U$
3. Intersection with $\emptyset$:	$A \cap \emptyset = \emptyset$
4. Complement of Union and $\emptyset$:	$U' = \emptyset$ $\emptyset' = U$
5. Every set is subset of Universal	$\forall A \in \{Sets\}, A \subseteq U$
6. Empty set is subset of every set	$\forall A \in \{Sets\}, \emptyset \subseteq A$
7. Definition of Empty Set	$\forall A \in \{Sets\}, A = \emptyset \leftrightarrow \forall x \in U, x \notin A$