Sample Problems for all sections of CMSC250, Midterm 1–Fall 2014

1. Translate each of the following English sentences into formal statements using the logical operators ($\forall, \exists, \vee, \wedge, \neg$, and $\rightarrow$). You may also use mathematical grouping and set notations, as needed. Under each translation that you provide, you will then write its negation as a *formal* statement (using the logical operators, domains and predicates).

   (a) English statement: "There exists exactly one integer e, such that $e + n = n$ for all integers n." Assume the domain $\mathbb{Z} = \{\text{all of the integers}\}$, .

   Symbolic restatement: ________________________________

   Its negation: ______________________________________

   (b) English statement: "No integer is both even and odd." Domain: $\mathbb{Z}$; Predicates: `Even`(x) and `Odd`x, for x being even or odd, respectively.

   Symbolic restatement: ________________________________

   Its negation: ______________________________________

2. (a) Complete the truth table for the following function: on input $b_2b_1b_0$, a 3 bit number, output 1 if the number is a prime, and output 0 if the number is not a prime. (Note that 0 and 1 are NOT primes).

b_2	b_1	b_0	
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

(b) Use the method given in class to obtain a Boolean Formula from this truth table. (Which is to say: construct a boolean expression in "disjunctive normal form," which also is how it was done in the text.) DO NOT SIMPLIFY.

(c) Draw a circuit for the formula you gave in part 2b. DO NOT SIMPLIFY. You may use AND, OR, and NOT gates only. The gates may have any number of inputs and any number of outputs.

3. Using 8-bit binary arithmetic. perform the following operation $25 - 91$. To simplify matters, you will perform your calculations in parts:

   (a) Write 25 (base-10) in base-2:

   (a) ____________

   (b) Write 91 (base-10) in base-2:

   (b) ____________

   (c) Convert the base-2 number from part(b) into its twos-complement form:

   (c) ____________

   (d) Perform the operation by summing the last two numbers, i.e., the numbers from part(b) and part(c); leave your answer in twos-complement form:

   (d) ____________

   (e) Using the twos-complement, rewrite the number you computed in part(d) as a positive base-2 integer:

   (e) ____________

4. Consider the following logical argument. (Note, **P** stands for proposition, and **C** stands for conclusion.)

P1	$\neg m \rightarrow \neg s$
P2	$\neg r \vee s$
P3	r
C	m

(a) Show, using a truth-table, that the argument is valid.

(b) Show, using the rules of inference that the argument is valid.

5. Prove or provide a counter example that
   (a) The sum of two consecutive integers is odd.

   (b) The product of three consecutive integers is even.

6. Give an irrational number x such that $10 \leq x \leq 11$.

COMMONLY USED LAWS, PROPERTIES & DEFINITIONS: LOGIC

CMSC 250

Logic

Given any statement variables p, q, and r, a tautology t and a contradiction c, the following logical equivalences hold:		
1. Commutative laws:	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. Associative laws:	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive laws:	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity laws:	$p \wedge t \equiv p$	$p \vee c \equiv p$
5. Negation laws:	$p \vee \neg p \equiv t$	$p \wedge \neg p \equiv c$
6. Double Negative law:	$\neg(\neg p) \equiv p$	
7. Idempotent laws:	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. DeMorgan's laws:	$\neg(p \wedge q) \equiv \neg p \vee \neg q$	$\neg(p \vee q) \equiv \neg p \wedge \neg q$
9. Universal bounds laws:	$p \vee t \equiv t$	$p \wedge c \equiv c$
10. Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. Negations of t and c:	$\neg t \equiv c$	$\neg c \equiv t$

Modus Ponens $p \to q$ p Therefore q	Modus Tollens $p \to q$ $\neg q$ Therefore $\neg p$		Disjunctive Syllogism	$p \vee q$ $\neg q$ Therefore p	$p \vee q$ $\neg p$ Therefore q
Conjunctive Addition	p q Therefore $p \wedge q$		Hypothetical Syllogism	$p \to q$ $q \to r$ Therefore $p \to r$	
Disjunctive Addition	p Therefore $p \vee q$	q Therefore $p \vee q$	Dilemma: Proof by Division into Cases	$p \vee q$ $p \to r$ $q \to r$ Therefore r	
Conjunctive Simplification	$p \wedge q$ Therefore p	$p \wedge q$ Therefore q	Rule of Contradiction	$\neg p \to c$ Therefore p	
Closing C.W. without contradiction	$\|p$ Assumed $\|q$ derived Therefore $p \to q$		Closing C.W. with contradiction	$\|p$ Assumed $\|x \wedge \neg x$ derived Therefore $\neg p$	