Name & UID: ____________________

Fall 2014 CMSC250/250H Midterm I

Circle Your Section!			
0101 (10am: 3120, Ladan)	0102 (11am: 3120, Ladan)	0103 (Noon: 3120, Peter)	
0201 (2pm: 3120, Yi)	0202 (10am: 1121, Vikas)	0203 (11am: 1121, Vikas)	0204 (9am: 2117, Karthik)
0301 (9am: 3120, Huijing)	0302 (8am: 3120, Huijing)	0303 (1pm: 3120, Yi)	250H (10am: 2117, Peter)

Do not open this exam until you are told. Read these instructions:

1. **This is a closed book exam.** No calculators, notes, or other aids are allowed. If you have a question during the exam, please raise your hand.
2. You must turn in your exam **immediately** when time is called at the end.
3. In order to be eligible for as much partial credit as possible, show all of your work for each problem, write legibly, and clearly indicate your answers. **Credit will not be given for illegible answers.**
4. After the last page there is paper for scratch work. If you need extra scratch paper after you have filled these areas up, please raise your hand. Scratch paper must be turned in with your exam, with your name and ID number written on it, but scratch paper will not be graded. You may also use the scratch paper as extra space for answers, but you must cross reference it.
5. You may not give or receive any unauthorized assistance on this examination.
6. **Write your name legibly on the top of each page of this exam.** Credit will not be given for lost/missing pages.

Question	1	2	3	4	5	6	7	8	9
Points	20	10	10	5	5	15	10	10	15
Score									

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1. (20 points) Define the following propositions:

- b= "I must wear a black shirt"
- r= "I must wear a red shirt"

...and the follow predicates:

- W(x)= "Shirt x is white"
- L(x)= "Shirt x is in the laundry"

In the problems that follow, assume the domain of discourse is my shirts.

Write the following sentences in formal logic using the above propositions and predicates. You may use the connectives $\{\wedge, \vee, \rightarrow, \neg\}$ and the quantifiers $\{\exists, \forall\}$. You may use the equals sign $\{=\}$ to denote that two shirts are the same.

(a) I have no white shirts in the laundry.

Solution:

$$\neg(\exists x)[W(x) \wedge L(x)]$$
$$(\forall x)[\neg(W(x) \wedge L(x))]$$
$$(\forall x)[\neg W(x) \vee \neg L(x)]$$
$$(\forall x)[W(x) \rightarrow \neg L(x)]$$
$$(\forall x)[L(x) \rightarrow \neg W(x)]$$

(b) There is exactly one white shirt in the laundry.

Solution:

$$(\exists s)[W(s) \wedge L(s)] \wedge ((\forall u)[W(u) \wedge L(u) \rightarrow u = s])$$
$$((\exists s)[W(s) \wedge L(s)]) \wedge ((\forall x, y)[W(x) \wedge L(x) \wedge W(y) \wedge L(y) \rightarrow x = y])$$

(c) If all of my white shirts are in the laundry, then I must wear a black shirt or a red shirt (or both).

Solution:

$$((\forall x)[W(x) \rightarrow L(x)]) \rightarrow (b \vee r)$$

(d) I don't have to wear a black shirt only if there is a white shirt that is not in the laundry.

Solution:

$$\neg b \rightarrow \exists x, W(x) \wedge \neg L(x)$$

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2. (10 points) Simplify the following expression. Show every step of your derivation, one step per line. **On each line, you must state the name of the logic law that was used to obtain that line from the last.** Use the handout containing all the logic laws.

$$(p \vee q \vee r) \wedge (p \vee q \vee \neg r) \wedge (\neg p \vee q) \wedge (r \vee q)$$

Solution:

	$(q \vee p \vee r) \wedge (q \vee p \vee \neg r) \wedge (\neg q \vee p) \wedge (q \vee r)$	Commutative law
$\equiv$	$q \vee [(p \vee r) \wedge (p \vee \neg r) \wedge \neg p \wedge r]$	Distributive law
$\equiv$	$q \vee [(p \vee (r \wedge \neg r)) \wedge \neg p \wedge r]$	Distributive law
$\equiv$	$q \vee [(p \vee F) \wedge \neg p \wedge r]$	Negation law
$\equiv$	$q \vee [p \wedge \neg p \wedge r]$	Universal bounds law
$\equiv$	$q \vee [F \wedge r]$	Negation law
$\equiv$	$q \vee F$	Universal bounds law
$\equiv$	q	Universal bounds law

3. (10 points) Simplify the following expression so that no negation appears to the left of a quantifier, and no negation appears outside of a parenthesis containing multiple predicates. In other words, move the negations "as far in" to the expression as possible. Write one step of your simplification per line. You do NOT need to state the law used to obtain each line.

$$\neg(\forall x, \exists y)[p(x) \wedge (\neg q(x) \vee r(x))]$$

Solution:

$$\begin{aligned}
& \neg(\forall x, \exists y)[p(x) \wedge (\neg q(x) \vee r(x))] \\
\equiv\ & (\exists x, \neg\exists y)[p(x) \wedge (\neg q(x) \vee r(x))] \\
\equiv\ & (\exists x, \forall y)\neg[p(x) \wedge (\neg q(x) \vee r(x))] \\
\equiv\ & (\exists x, \forall y)\neg p(x) \vee \neg(\neg q(x) \vee r(x))] \\
\equiv\ & (\exists x, \forall y)\neg p(x) \vee (q(x) \wedge \neg r(x))]
\end{aligned}$$

4. (5 points) Using only AND, OR, and NOT gates that take up two inputs, draw a circuit that outputs z from input wires x and y:

$$z = (x \land \neg y) \lor (y \land \neg x)$$

Solution: OMITTED

5. (5 points) Using disjunction normal form, write a logical expression for s, which is defined by the following truth table. Do not simplify.

p	q	r	s
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	T

Solution:

$$(p \land \neg q \land r) \lor (p \land \neg q \neg \land r) \lor (\neg p \land \neg q \land \neg r)$$

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6. (a) (15 points) Write the base-6 number $(205)_6$ in binary (base-2) form. Show your work.

Solution: First convert to decimal

$$(205)_6 = 2 * 36 + 5 = 77$$

Then make it binary...

$$77 = 2 * 38 + 1$$

$$38 = 2 * 19 + 0$$

$$19 = 2 * 9 + 1$$

$$9 = 2 * 4 + 1$$

$$4 = 2 * 2 + 0$$

$$2 = 2 * 1 + 0$$

$$1 = 2 * 0 + 1$$

Final answer: 1001101

(b) Write the binary number 100101001011100 in hexadecimal form.

Solution: 4A5C

(c) The following number is written using an 8-bit **two's complement** representation. Write the number in base 10. Include a "+" or "−" sign in front of your answer. Show your work.

11110101

Solution: First, subtract 1 to get the one's complement:

11110100.

Next, flip the bits to get the original number

00001011.

In base ten this is 11. But the original number was negative (first bit was a 1), so the answer is -11.

7. (a) (10 points) State the definition of "odd" as given in class and in the book.

Solution: Even when $\exists m \in Z, n = 2m$, odd when $\exists m \in Z, n = 2m + 1$.

(b) Suppose that a and b are odd. Prove that $a+3b$ is even. You must use the basic definitions of even and odd (as in part a). You may NOT use the fact that the sum of an even and odd number is odd.

Solution: Let $a = 2m + 1$ and $b = 2n + 1$ for $m, n \in \mathbb{Z}$ (by the definition of odd). So,

$$\begin{array}{rcll} a + 3b & = & (2m+1) + (3(2n+1)) & \text{by substitution} \\ & = & 2m + 1 + 6n + 3 & \text{by algebra} \qquad ll \\ & = & 2(m + 3n + 2) & \text{by algebra} \end{array}$$

which is even by the definition of even and the fact that the integers are closed under addition and multiplication.

8. (10 points) I have a bag containing only black marbles and white marbles. I draw three marbles from the bag, which I call marble 1, marble 2, and marble 3. Consider the following propositions:

   - W_i: "Marble i is white"
   - B_i: "Marble i is black"

   For example, W_2 is the proposition "Marble 2 is white" and B_3 is the proposition "Marble 3 is black." The compound proposition $W_1 \wedge B_2$ reads "Marble 1 is white and marble 2 is black."

   Write a compound proposition that is true (satisfied) if and only if the following holds: "At most two of the drawn marbles are white, and at most two are black."

   You may use only the atomic propositions W_1, W_2, W_3, B_1, B_2, B_3, and the logical operators $\{\neg, \vee, \wedge\}$. You may NOT use quantifiers.

Solution:

$(W_1 \vee B_1) \wedge (W_2 \vee B_2) \wedge (W_3 \vee B_3)$	All marbles have a color
$\neg(W_1 \wedge B_1) \wedge \neg(W_2 \wedge B_2) \wedge \neg(W_3 \wedge B_3)$	No marble has two colors
$\wedge\neg(W_1 \wedge W_2 \wedge W_2)$	AND not all marbles are white
$\wedge\neg(B_1 \wedge B_2 \wedge B_2)$	AND not all marbles are black

9. (15 points) In this problem you may assume that $\sqrt{2}$, π, and e are irrational. You may also use the following facts: (a) The sum of a rational and an irrational is irrational, (b) the product of an irrational and a non-zero rational is irrational, (c) an irrational divided by a non-zero rational is irrational, (d) both the integers and rationals are closed under multiplication and addition.

   (a) Prove there exists two irrational numbers, a and b, such that $\frac{a+b}{2}$ is rational.

   Solution: $a = \sqrt{2}, b = -\sqrt{2}$

   (b) Give a constructive proof of the following statement: **Between any two irrational numbers, a and b, there is another irrational number.** You must prove that your solution does indeed lie between a and b.

   Solution: There are many ways to do this.

   ALTERNATIVE.
   Let the numbers be a, b with $a < b$. Consider the number $c = (a+b)/2$. If this number is irrational, then we're done. If not, then consider the number $(a+c)/2$. This is an average of an irrational and a rational number, which is irrational.

   ALTERNATIVE: Find a rational smaller than $b - a$ (and then add it to a). Find n such that

$$\frac{1}{n} \leq \frac{b-a}{2} \implies n \geq \frac{2}{b-a}$$

   So $n = \left\lceil \frac{2}{b-a} \right\rceil$ suffices. The solution is then $a + \frac{1}{n}$.

SCRATCH PAPER

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