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Fall 2014 CMSC250/250H Practice Problems II

Circle Your Section!			
0101 (10am: 3120, Ladan)	0102 (11am: 3120, Ladan)	0103 (Noon: 3120, Peter)	
0201 (2pm: 3120, Yi)	0202 (10am: 1121, Vikas)	0203 (11am: 1121, Vikas)	0204 (9am: 2117, Karthik)
0301 (9am: 3120, Huijing)	0302 (8am: 3120, Huijing)	0303 (1pm: 3120, Yi)	250H (10am: 2117, Peter)

Do not open this exam until you are told. Read these instructions:

1. **Write your name legibly on the top of each page of this exam.** Credit will not be given for lost/missing pages.
2. **CIRCLE YOUR SECTION**
3. **This is a closed book exam.** No calculators, notes, or other aids are allowed. If you have a question during the exam, please raise your hand.
4. You must turn in your exam **immediately** when time is called at the end.
5. In order to be eligible for as much partial credit as possible, show all of your work for each problem, write legibly, and clearly indicate your answers. **Credit will not be given for illegible answers.**
6. After the last page there is paper for scratch work. If you need extra scratch paper after you have filled these areas up, please raise your hand. Scratch paper must be turned in with your exam, with your name and ID number written on it, but scratch paper will not be graded. You may also use the scratch paper as extra space for answers, but you must cross reference it.
7. You may not give or receive any unauthorized assistance on this examination.

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1. For each function, write whether it is an injection, surjection, bijection, or neither from R to R . If it is not a bijection, write down a domain and range over which the function IS a bijection.

(a) $f(x) = x^2$

(b) $f(x) = x^3$

(c) $f(x) = \sqrt{x-2}$

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2. (10 points)

(a) Which has larger cardinality: Z^2 or $(Z \times R)$?

(b) Prove that the sets $A = \{2k-3 | k \geq 0 \in Z\}$ and $B = \{k^2 | k \in Z\}$ have the same cardinality by writing down a bijection from A to B .

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3. (a) Evaluate: $\prod_{i=1}^{\infty} 2^{\frac{1}{2^i}}$. Your answer should be a single number. You may use known formulas to evaluate sums.

- (b) Write the following sum as a closed-form expression:

$$\sum_{i=-2}^{100} \frac{5^i + i + 1}{7}.$$

Do not simplify. You may use known formulas to evaluate sums.

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4. Solve your favorite induction problem from the induction handout or from class. Write only one step per line, and place a justification to the right of each step (ex: “By algebra” or “by induction hypothesis”).

(a) Base Case:

(b) Inductive Hypothesis:

(c) Inductive Step (label where you use the induction hypothesis):

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5. Solve your least favorite induction problem. Write only one step per line, and place a justification to the right of each step (ex: “By algebra” or “by induction hypothesis”).

(a) Base Case:

(b) Inductive Hypothesis:

(c) Inductive Step (label where you use the induction hypothesis):

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6.

(a) Prove the distributive law

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

You MUST use element-chasing. Your proof must be in two-column format with a justification for each step on the right.

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(b) For two sets A and B , prove:

$$\overline{(A \cup B)} \cap A = \emptyset.$$

Your proof MUST use the set identities (see handout). Your proof must be in two-column format with a justification for each step on the right.

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7. (10 points) Prove that $\sqrt{6}$ is irrational. Do NOT use the unique factorization theorem. Your proof must be in two-column format with a justification for each step on the right. If you use any lemmas, you must prove them using modular arithmetic.

8. (10 points) Prove that $\log_2 6$ is irrational. Use the unique factorization theorem. Your proof must be in two-column format with a justification for each step on the right.

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