

CMSC250H Honors Project

Fall 2014

You will write a short report on an important/classical math problem of your choosing. I have listed below the “extra credit” problems given in class, which I feel are good topics. However, you may choose any interesting problem from mathematics or computer science provided you ask for approval.

Your report **MUST** be typed and double-spaced. It should be approximately 3 pages long. Grades will be based on the following criteria: (1) spelling and grammar, (2) your explanation and understanding of the problem, and (3) how thoroughly you address related issues (such as progress on solving the problem, or the relation of the problem to other areas of mathematics).

The project is due in class on **Tuesday, December 9th**.

Extra-Credit Problems

- (1) Twin prime conjecture: Are there infinitely many twin primes?
 - Why is this problem so famous?
 - What progress has been made in this area? What similar things have been proven?
 - Have people tried to check the conjecture with computers? Why does this fail?
 - Are there other similar conjectures in number theory that were thought to be true (and “checked” for large numbers with computers), but turned out to be false?
 - Do you think the conjecture is true? Why?
- (2) Continuum Hypothesis: Does there exist a set with cardinality bigger than $\aleph_0$ but smaller than $\aleph_1$?
 - Does such a set exist or not?
 - What other problems are similar?
 - What does this say about the strength of mathematics?
 - What does this have to do with the Godel incompleteness theorems?
- (3) How hard is it to break RSA? Is it as hard as factoring?
 - Why is this problem important?
 - What classical mathematical hypothesis is closely related to this problem?
 - What are the best known algorithms for solving this problem? How good (or bad) are they?
- (4) 4-coloring Theorem
 - Explain the theorem
 - What other classical mathematical problems were solved by similar methods? (Hint: One example is the Kepler Conjecture)
 - What is surprising about the proof of this theorem? Why is this result so hard to prove?
 - Why do some mathematicians reject the proof?
 - Do you reject the proof? Why?
- (5) Godel Incompleteness Theorems
 - Explain the theorems
 - What do these theorems say about the power of mathematics?
 - Are there examples of mathematics statement that we cannot prove or disprove?
 - What does this say about Hilbert’s Program?

(6) Axiom of Choice

- Explain the axiom
- Is the axiom true or false?
- Why is this axiom controversial?
- Do most people accept the axiom?
- If you choose to accept it, what else do you need to accept? (hint: Banach-Tarski paradox)

Some other topic suggestions: NP-Completeness, non-computable functions, discrete logarithm problems, automated theorem proving.