VALID ARGUMENTS

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- Socrates is a man
- Men are mortal

$\rightarrow \quad \therefore$ Socrates is mortal

Premises
Hypothesis

Conclusions

VALID $\neq$ TRUE

Socrates is a man Men are mortal	$\rightarrow$	$\therefore$ Socrates is mortal	Valid argument Valid conclusion
Socrates is a man Men are mortal	$\rightarrow$	$\therefore$ Socrates is immortal	Invalid argument Invalid conclusion
Socrates is a man Men are immortal	$\rightarrow$	$\therefore$ Socrates is mortal	Invalid argument Valid conclusion
Socrates is a man Men are immortal	$\rightarrow$	$\therefore$ Socrates is immortal	Valid argument Invalid conclusion

MODUS PONENS

Socrates is a man

If Socrates is a man, then Socrates is mortal

$\longrightarrow$ $\therefore$ Socrates is mortal

p: Socrates is a man
q: Socrates is mortal

$$
\begin{array}{rl}
p \rightarrow q & \text{premise} \\
p & \text{premise} \\
\therefore q & \text{conclusion}
\end{array}
$$

Modus Ponens

SYLLOGISMS

Syllogistic:

$$\begin{array}{rl} p \to q & \text{premise} \\ p & \text{premise} \\ \therefore q & \text{conclusion} \end{array}$$

Modus Ponens

Tautological:

$$(p \wedge (p \to q)) \to q$$

MODUS TOLLENS

Syllogistic:

$$p \to q \text{ premise}$$
$$\neg q \text{ premise}$$
$$\therefore \neg p \text{ conclusion}$$

Modus Tollens

Tautological:

$$(\neg q \wedge (p \to q)) \to q$$

Prove on board

RULES OF INFERENCE

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$[p \land (p \to q)] \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$[\neg q \land (p \to q)] \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \land (q \to r)] \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$[(p \lor q) \land \neg p] \to q$	Disjunctive syllogism
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification
p q $\therefore p \land q$	$[(p) \land (q)] \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$[(p \lor q) \land (\neg p \lor r)] \to (q \lor r)$	Resolution

REAL-WORLD APPLICATION: WHERE ARE MY GLASSES?

(a) If I was reading the newspaper in the kitchen, my glasses are on the kitchen table
(b) If my glasses are on the kitchen table, then I saw them at breakfast
(c) I did not see my glasses at breakfast
(d) I was reading in the living room or the kitchen
(e) If I was reading in the living room, my glasses are on the coffee table

FALLACIES

$$p \to q$$

$$\begin{aligned} p \to q &\text{ premise} \\ q &\text{ premise} \\ \therefore p &\text{ WRONG!} \end{aligned}$$

Affirming the conclusion

$$\begin{aligned} p \to q &\text{ premise} \\ \neg p &\text{ premise} \\ \therefore \neg q &\text{ WRONG!} \end{aligned}$$

Denying the hypothesis

EXAMPLES

If Bill Gates owns Microsoft, then he is rich.
Bill Gates is rich.
∴ Bill Gates owns Microsoft

If Queen Elizabeth is American, she is not a space alien.
Queen Elizabeth is not American.
∴ Queen Elizabeth is a space alien

RIPPED FROM THE HEADLINES!

If Obama’s dad is American, then Obama was born in America
Obama’s dad is not American
$\therefore$ Obama was not born in America

If a child becomes autistic, then they’ve had the MMR vaccine
A child has the MMR vaccine
$\therefore$ A child will become autistic

FALLACIES *LOOK* OK

$$\begin{aligned} p \to q &\ \text{premise} \\ q &\ \text{premise} \\ \therefore p &\ \text{WRONG!} \end{aligned}$$

$$\begin{aligned} p \to q &\ \text{premise} \\ p &\ \text{premise} \\ \therefore q &\ \text{Valid} \end{aligned}$$

$$\begin{aligned} p \to q &\ \text{premise} \\ \neg p &\ \text{premise} \\ \therefore \neg q &\ \text{WRONG!} \end{aligned}$$

$$\begin{aligned} p \to q &\ \text{premise} \\ \neg q &\ \text{premise} \\ \therefore \neg p &\ \text{Valid} \end{aligned}$$

QUANTIFIERS

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	***Name***
$\dfrac{\forall x\, P(x)}{\therefore\ P(c)}$	Universal instantiation
$\dfrac{P(c) \text{ for an arbitrary } c}{\therefore\ \forall x\, P(x)}$	Universal generalization
$\dfrac{\exists x\, P(x)}{\therefore\ P(c) \text{ for some element } c}$	Existential instantiation
$\dfrac{P(c) \text{ for some element } c}{\therefore\ \exists x\, P(x)}$	Existential generalization

EXAMPLE 1

TABLE 2 **Rules of Inference for Quantified Statements.**

Rule of Inference	*Name*
$\dfrac{\forall x\, P(x)}{\therefore\ P(c)}$	Universal instantiation
$\dfrac{P(c) \text{ for an arbitrary } c}{\therefore\ \forall x\, P(x)}$	Universal generalization
$\dfrac{\exists x\, P(x)}{\therefore\ P(c) \text{ for some element } c}$	Existential instantiation
$\dfrac{P(c) \text{ for some element } c}{\therefore\ \exists x\, P(x)}$	Existential generalization

- No polynomial has an asymptote
- A function, f(x), has a horizontal asymptote

Prove: f(x) is not a polynomial

EXAMPLE 2

TABLE 2 **Rules of Inference for Quantified Statements.**

Rule of Inference	*Name*
$\dfrac{\forall x P(x)}{\therefore\ P(c)}$	Universal instantiation
$\dfrac{P(c) \text{ for an arbitrary } c}{\therefore\ \forall x P(x)}$	Universal generalization
$\dfrac{\exists x P(x)}{\therefore\ P(c) \text{ for some element } c}$	Existential instantiation
$\dfrac{P(c) \text{ for some element } c}{\therefore\ \exists x P(x)}$	Existential generalization

- A student has not read the book.
- Every student passed the exam
- ∴ Someone who passed the first exam has not read the book

EXAMPLE 3

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	*Name*
$\dfrac{\forall x\, P(x)}{\therefore\ P(c)}$	Universal instantiation
$\dfrac{P(c) \text{ for an arbitrary } c}{\therefore\ \forall x\, P(x)}$	Universal generalization
$\dfrac{\exists x\, P(x)}{\therefore\ P(c) \text{ for some element } c}$	Existential instantiation
$\dfrac{P(c) \text{ for some element } c}{\therefore\ \exists x\, P(x)}$	Existential generalization

$$\forall x, P(x) \rightarrow Q(x)$$
$$\forall x, P(x)$$
$$\therefore \forall x, Q(x)$$