Name & UID: ______________________________

Circle Your Section!			
0101 (10am: 3120, Ladan)	0102 (11am: 3120, Ladan)	0103 (Noon: 3120, Peter)	
0201 (2pm: 3120, Yi)	0202 (10am: 1121, Vikas)	0203 (11am: 1121, Vikas)	0204 (9am: 2117, Karthik)
0301 (9am: 3120, Huijing)	0302 (8am: 3120, Huijing)	0303 (1pm: 3120, Yi)	250H (10am: 2117, Peter)

1. (40 points) Translate the following English sentences into formal language (statements using quantifiers and propositions). Then write the negation of the original sentence in plain English, and translate the negation into formal language. Make sure all negations come after all of the quantifiers. Make your English sentences and formal logic statements clean and neat. You may use symbols such $\forall, \exists, \in$ and mathematical and logical symbols. Let the predicate $T(s, c) =$ "student s has taken computer science course c", where the domain of discourse it $S = \{\text{Students}\}$, and $C = \{\text{Computer science courses at the University of Maryland}\}$.

   (a) Joseph has taken cmsc250.

   (b) Joseph has taken all computer science courses at Maryland.

   (c) All students at Maryland have taken some computer science course.

   (d) There are two students at Maryland such that one of them has taken all of the computer science courses the other has taken.

2. (30 points) Consider the following statements:

   1. Either Barack is president or Mitt is president.
   2. If Michelle is not first lady and Barack is president, then arugula is served for dinner tonight.
   3. Mitt Romney is not president.
   4. Arugula is never served on any night in the white house.

   Define the following propositions:

   - BP = Barack is president
   - MP = Mitt is president
   - MF = Michelle is first lady

   ...and the following predicate

   - AS(x) = Arugula is served at the white house on night "x."

   Prove using a formal argument that Michelle is the first lady. For every step of your argument, you must state a reason/justification. Your justifications may be drawn from either the logical equivalences (page 27, table 6 in Rosen), the rules of inference (page 72, table 1, in Rosen), or the rules of inference for quantifiers (page 76, table 2 in Rosen). These tables are included on page 4 of this assignment.

3. (30 points) A *square-free 3-coloring of* $1, \ldots, n$ is an assignment of Red, Blue, or Green to the integers $1, \ldots, n$ so that no two integers exactly a perfect square apart have the same color. Let $R(i)$, $B(i)$, and $G(i)$ be predicates representing that number i is colored Red, Blue, or Green, respectively.

   Write a quantified statement that is true if and only if a square-free 3-coloring of $1, \ldots, 16$ exists. You may use $\{a, a+1, \ldots, b\}$ to represent the set of integers from a to b. Your statement should not index R, B, G outside the domain $\{1, 2, \ldots, 16\}$. You may have several quantified statements ANDed together.

   NOTE: You can imagine a computer language that has an instruction:

   Set the values $R(i), B(i), G(i)$ such that your quantified statement is true.

   This would be like a SAT solver with a much more powerful language.

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \wedge (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \wedge (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \to q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Simplification
p q $\therefore p \wedge q$	$((p) \wedge (q)) \to (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$	Resolution

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	*Name*
$\forall x P(x)$ $\therefore P(c)$	Universal instantiation
$P(c)$ for an arbitrary c $\therefore \forall x P(x)$	Universal generalization
$\exists x P(x)$ $\therefore P(c)$ for some element c	Existential instantiation
$P(c)$ for some element c $\therefore \exists x P(x)$	Existential generalization