CMSC250 Homework 7 Due: Wednesday, November 5, 2014

Name & UID: ______________________________

Circle Your Section!			
0101 (10am: 3120, Ladan)	0102 (11am: 3120, Ladan)	0103 (Noon: 3120, Peter)	
0201 (2pm: 3120, Yi)	0202 (10am: 1121, Vikas)	0203 (11am: 1121, Vikas)	0204 (9am: 2117, Karthik)
0301 (9am: 3120, Huijing)	0302 (8am: 3120, Huijing)	0303 (1pm: 3120, Yi)	250H (10am: 2117, Peter)

Question:	1	2	3	4	5	6	Total
Points:	10	20	20	30	10	10	100
Score:							

1. (10 points) Show that if $7|n^2$ then $7|n$ (without using the Unique Factorization Theorem).

2. (20 points) Show that $\sqrt{7}$ is irrational.

   (a) Use the Unique Factorization Theorem.

   (b) Do not use the Unique Factorization Theorem.

3. (20 points) Consider the sequence $w_0, w_1, w_2, \ldots$, where w_i is the number of 1's in the binary representation of i.

   (a) Write down the first thirteen terms of the sequence (i.e., $w_0, w_1, w_2, \ldots, w_{12}$). No justification needed.

   (b) Calculate $\sum_{i=0}^{12} w_i$. No justification needed.

4. (30 points) For each sequence, write a simple, closed-form expression for the ith term. HINT: It can sometimes be helpful look at the difference of consecutive terms ($a_{i+1} - a_i$) or the ratio of consecutive terms (a_{i+1}/a_i) to see what is happening.

   (a) $-\frac{1}{2}, \frac{1}{5}, -\frac{1}{8}, \frac{1}{11}, -\frac{1}{14}, \ldots,$

   (b) 3, 5, 11, 29, 83, ...,

   (c) 2, -3, 6, -13, 28, -59, ...,

5. (10 points) Give an explicit mapping to show that the postive, even integers ($\mathbb{Z}^{\text{even},+}$) have the same cardinality as the integers divisible by 3 ($\{n$ such that $3|n\}$). You should produce a function that is a bijection. It should be an explicit formula not involving cases.

6. (10 points) Find all real numbers x satisfying

$$\lfloor x+1 \rfloor = \lfloor 4x \rfloor \ .$$

Show your work. HINT: Every real number x can be written as $n+\epsilon$ where n is an integer and ϵ is a real number such that $0 \leq \epsilon < 1$.