Name & UID: ________________________________

Circle Your Section!			
0101 (10am: 3120, Ladan)	0102 (11am: 3120, Ladan)	0103 (Noon: 3120, Peter)	
0201 (2pm: 3120, Yi)	0202 (10am: 1121, Vikas)	0203 (11am: 1121, Vikas)	0204 (9am: 2117, Karthik)
0301 (9am: 3120, Huijing)	0302 (8am: 3120, Huijing)	0303 (1pm: 3120, Yi)	250H (10am: 2117, Peter)

Question:	1	2	3	4	5	Total
Points:	20	20	20	20	20	100
Score:						

1. (20 points) Let $P(n)$ be a predicate. Assume you have proven that $P(9)$ holds and that $P(n-2) \rightarrow P(n+3)$. What do you know? State your answer simply and clearly using modular arithmetic. No justification needed.

2. (20 points) We will use Mathematical Induction to prove that for $n \geq 2$

$$\sum_{j=2}^{n} \frac{1}{(j-1)j} \;=\; 1 - \frac{1}{n}$$

(a) Prove the base case.

(b) What is the Inductive Hypothesis?

(c) Show the Inductive Step.

3. (20 points) We will use Mathematical Induction to prove that for $n \geq 1$

$$\sum_{i=1}^{n} i(i!) \ = \ (n+1)! - 1$$

(a) Prove the base case.

(b) What is the Inductive Hypothesis?

(c) Show the Inductive Step.

4. (20 points) We will use Mathematical Induction to show that, for all integers $n \geq 1$, $8|(3^{2n} - 1)$.
   (a) Prove the base case.

   (b) What is the Inductive Hypothesis?

   (c) Show the Inductive Step.

5. (20 points) We will use Mathematical Induction to show that, for all integers $n \geq 2$, if $x_1, x_2, ..., x_n$ are real numbers strictly between 0 and 1 (i.e., $0 < x_i < 1$), then

$$(1 - x_1)(1 - x_2) \cdots (1 - x_n) > 1 - x_1 - x_2 - \cdots - x_n$$

   (a) Prove the base case.

   (b) What is the Inductive Hypothesis?

   (c) Show the Inductive Step.