These are practice problems for the upcoming final exam. **Warning:** This does not necessarily reflect the length, difficulty, or coverage of the actual exam.

1. General knowledge of sets, functions, and number systems.

   (a) T F All countable sets are finite.

   (a) ______________

   (b) T F In a proof by contradiction, we first assume that the statement we are trying to prove is false.

   (b) ______________

   (c) T F There exists a bijection between the integers and the real numbers.

   (c) ______________

   (d) T F The null set is an element of the power set of any set.

   (d) ______________

   (e) T F The combination of 26 items taken three at a time is the number of unique 3-letter strings that can be constructed using the English alphabet.

   (e) ______________

   (f) T F Let f and g be two functions with domain and codomain D. Then $f \circ g(x) = g \circ f(x)$ for all $x \in D$.

   (f) ______________

Fill the Short Blanks

2. Provide an appropriate answer to each of the following questions.

   (a) Give the contrapositive of the following statement
   If you enjoyed CMSC 250 then you will enjoy CMSC 451.

   (b) For how many rows of the truth table is the following formula TRUE? (All we want is a numeric answer. You may do a truth table but it will not be graded.)
   $(x \vee y \vee \sim z) \wedge (\sim x \vee \sim y \vee \sim z)$

(c) Consider the two quantified logic expressions
A: $(\forall x)(\exists y)P(x, y)$
and
B: $(\exists y)(\forall x)P(x, y)$
Circle whether **A** is **True** or **False**, and whether **B** is **True** or **False** each of the following interpretations (DO NOT JUSTIFY). (Suggestion: Do all of **A** then all of **B**.)

1. the domain is $\mathbb{R}$ and $P(x, y)$ means $x > y$ **A: True/ False** **B: True/ False**.
2. the domain is $\mathbb{R}$ and $P(x, y)$ means $x \geq y$ **A: True/ False** **B: True/ False**.
3. the domain is $\mathbb{N}$ and $P(x, y)$ means $x > y$ **A: True/ False** **B: True/ False**.
4. the domain is $\mathbb{N}$ and $P(x, y)$ means $x \geq y$ **A: True/ False** **B: True/ False**.

(d) The Big Ten currently comprises 14 schools. How many student athletes must be present at a brunch to guarantee that at least 6 represent the same school? Use the Pigeon Hole principle and assume that each school has many student athletes.

Simply Weak Induction

Prove by **weak** mathematical induction that:

$$\forall n \geq 12 \quad \sum_{k=12}^{n} k = \frac{(n-11)(n+12)}{2}\ .$$

Base Case:

Induction Hypothesis: You must state it in the standard way for weak induction, i.e., do not write as if this is a strong induction proof.

Inductive Step: Justify every step.

3. Prove that $\sqrt[7]{3}$ is irrational. You may use the fact that if $3|n^7$ then $3|n$. You may **not** use the Unique Factorization Theorem. Justify every step (of your proof).

4. Use the Unique Factorization Theorem to prove that $\sqrt[7]{3}$ is irrational. Justify every step of your proof.

5. Kruskal Kards is played using a K-deck, produced by adding a fifth suit–the Kruskals– with the same 13 denominations, Ace through King, to a standard 52-card deck.

   KV poker, a version of 5-card poker where players can score with 5-of-a-kind and a Royal straight Kruskal (A-K-Q-J-10 in the K-suit) in addition to the standard poker hands.

   Do NOT simplify your answers–products and quotients of combinations, permutations, and factorials are just fine.

   (a) Number of unique 5 card hands:

   (b) Number of possible 5-of-a-kind hands:

   (c) Number of hands with exactly 4-of-a-kind:

   (d) Number of hands where all 5 cards are in the same suit (flush):

   (e) Number of hands which contain three of one denomination and two of another (full house):

   (f) Number of hands which contain two of one denomination, two of another denomination, and another card of neither denomination (two pair):

   (g) Number of hands which are **not** a Royal Straight Kruskal:

Floored Functions, Some Ceilings

For each of the following questions say YES or NO and prove your assertion. (Recall that $\mathbb{R}$ is the Reals and $\mathbb{Z}$ is the Integers.)

6. Let $f : \mathbb{R} \times \mathbb{R} \to \mathbb{Z}$ be defined by $f(x, y) = \lfloor x \rfloor + \lfloor y \rfloor$.
   (a) Is f 1-1? Justify your answer.

   (b) Is f onto? Justify your answer.

7. Let $f : \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ be defined by $f(x) = (\lfloor x \rfloor, \lceil x \rceil)$.
   (a) Is f 1-1? Justify your answer.

   (b) Is f onto? Justify your answer.

Function Junction

8. Let A and B be finite sets. Let a be the size of A. Let b be the size of B. Assume $0 < a < b$. Answer the following questions. No proof required. HINT: It is helpful to draw an example ("external") diagram of a small function to see what is happening.

   (a) How many functions are there with domain A and co-domain B?

   (b) How many one-to-one functions are there with domain A and co-domain B?

   (c) How many one-to-one functions are there with domain B and co-domain A? (NOTE- domain is B, co-domain is A.)

   (d) How many onto functions are there with domain A and co-domain B?