Summations and Products

Sequences

- $2, 4, 6, 8, \ldots$ for $i \geq 1$ $a_i = 2i$
  - an infinite sequence
- for $i \geq 1$ $b_i = (-1)^i$
  - an infinite sequence
- for $1 \leq i \leq 6$ $c_i = i + 5$
  - a finite sequence

Finding an explicit formula

- Figure out the formula for this sequence:

$$1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, \ldots$$

A Familiar Sequence

- Put two points on a circle; draw a line between them. How many regions?
- Put three points on a circle; draw a line between all pairs of points. How many regions?
- Put four points on a circle; draw a line between all pairs of points. How many regions?

What is the pattern?

Another Familiar Sequence

$$k \geq 0:$$

$$a_k = 2k + 1$$

$$b_k = (k-1)^3 + k + 2$$

Summation & product notation

- Sum of items specified

$$\sum_{k=1}^{6} 2^k = 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6$$

- Product of items specified

$$\prod_{k=1}^{5} 2k = (2 \cdot 1)(2 \cdot 2)(2 \cdot 3)(2 \cdot 4)(2 \cdot 5)$$

Variable ending point

- *n* as the index of the final term

$$\sum_{k=0}^{n} \frac{k+1}{n+k}$$

- for $n = 2$
- for $n = 3$

Telescoping series

$$\sum_{k=1}^{n}\left(\frac{k}{k+1}-\frac{k+1}{k+2}\right)$$

$$\prod_{i=1}^{n}\frac{i}{i+1}$$

Properties

- Merging and splitting

$$\sum_{k=m}^{n} a_k + \sum_{k=m}^{n} b_k = \sum_{k=m}^{n} (a_k + b_k)$$

$$\sum_{k=m}^{n} a_k = \sum_{k=m}^{i} a_k + \sum_{k=i+1}^{n} a_k$$

Properties, con't.

- Distribution

$$c\sum_{k=m}^{n} a_k = \sum_{k=m}^{n} c a_k$$

Properties, con't.

- Distribution

$$\left(\sum_{i=1}^{m} a_i\right)\left(\sum_{j=1}^{n} b_j\right) = \sum_{i=1}^{m}\left(\sum_{j=1}^{n} a_i b_j\right)$$

Properties, con't.

- Interchanging order of summation

$$\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} = \sum_{j=1}^{n} \sum_{i=1}^{m} a_{ij}$$

Nesting of sum/product notation

- Variations (same or different??):

$$\sum_{i=1}^{m}\sum_{j=1}^{n}Y_{ij}{}^{2} \qquad \sum_{i=1}^{m}(\sum_{j=1}^{n}Y_{ij})^{2} \qquad (\sum_{i=1}^{m}\sum_{j=1}^{n}Y_{ij})^{2}$$

Factorial

- Product definition:

$$n! = \prod_{i=1}^{n} i = n(n-1)(n-2) \ldots 2 \bullet 1$$

- Recursive definition:

$$0! = 1$$

$$n! = n(n-1)!$$