Weak Induction

1. (Problem 3, Page 329.) Use Mathematical Induction to prove:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

2. (Problem 4, Page 329.) Use Mathematical Induction to prove:

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

3. Use Mathematical Induction to prove:

$$\prod_{i=2}^{n}(1 - \frac{1}{i}) = \frac{1}{n}$$

4. (Problem 27, Page 330.) Use Mathematical Induction to show that for $n \geq 1$

$$\sum_{i=1}^{n} \frac{1}{\sqrt{i}} > 2(\sqrt{n+1} - 1)$$

5. (Problem 45, Page 331.) Use mathematical Induction to prove that a set with n elements has $n(n-1)/2$ subsets containing exactly two elements.

6. (Problem 46, Page 331.) Use mathematical Induction to prove that a set with n elements has $n(n-1)(n-2)/6$ subsets containing exactly three elements.

7. (Problem 60, Page 332.) Use mathematical induction to show that for propositions p_i $(1 \leq i \leq n)$

$$\neg(p_1 \vee p_2 \vee \cdots \vee p_n) \equiv \neg p_1 \wedge \neg p_2 \wedge \cdots \wedge \neg p_n \ .$$

Strong Induction

1. (Problem 6, Page 342)
   (a) Determine which amounts of postage can be formed using just 3-cent and 10-cent stamps.
   (b) Prove your answer using Weak Mathematical Induction.
   (c) Prove your answer using Strong Mathematical Induction.

2. Consider the following recurrence where $a_0 = 1$.

$$a_n \;=\; a_{n-1} + 2a_{n-2} + 4a_{n-3} + 8a_{n-4} + \cdots + 2^{n-1}a_0 \;.$$

   (a) Rewrite the recurrence with the right side as a summation.
   (b) Evaluate a_n for $n = 0, 1, 2, 3, 4, 5$.
   (c) Write a simple formula for a_n.
   (d) Prove that your formula for a_n is correct using Weak Mathematical Induction.
   (e) Prove that your formula is correct using Strong Mathematical Induction (ignoring the base case which has already been proved).

3. (Theorem 1, Page 339) Prove that a simple polygon with n sides, where n is an integer with $n \geq 3$, can be triangulated into $n - 2$ triangles. [Use the lemma from the next problem.]

4. (Lemma 1, Page 339) Prove that every simple polygon with at least four sides has an interior diagonal.

5. (Problem 19, Page 342) Pick's Theorem.

Constructive Induction

1. Assume that you guess that $\sum_{i=1}^{n} i^2$ is a cubic polynomial. Use Constructive Induction to derive the exact polynomial.

2. Use Constructive Induction to find an upper bound on F_n in the recurrence

$$F_n = F_{n-1} + 3F_{n-2}$$

   where $F_0 = 1$ and $F_1 = 2$.