1. We would like to figure out if there is something special about the product of four consecutive integers.
   (a) Calculate (the early ones by hand) $0 \cdot 1 \cdot 2 \cdot 3$, $1 \cdot 2 \cdot 3 \cdot 4$, $2 \cdot 3 \cdot 4 \cdot 5$, $3 \cdot 4 \cdot 5 \cdot 6$, ..., until a pattern emerges.
   (b) Prove your answer.

2. (a) Write 6! in standard factored form.
   (b) Write 20! in standard factored form.
   (c) Without computing the value of 20! determine how many zeros are at the end of this number when written in decimal form. Justify your answer.
   (d) Without computing the value of 30! determine how many zeros are at the end of this number when written in decimal form. Justify your answer.

3. Show that two consecutive integers cannot both be odd.

4. Show that the product of two consecutive integers is even.

5. Prove by contradiction that the product of a nonzero rational number and an irrational number is irrational.

6. Prove that there is a rational number between any to rationals.

7. Prove that least one of the real numbers $a_1, a_2, \ldots, a_n$ is greater than or equal to the average of these numbers. What kind of proof did you use? [Page 92, Problem 39. Answer Page S-10.]

8. Use the previous question to show that if the first ten positive integers are placed around a circle in any order, there exist three integers in consecutive locations around the circle that have a sum greater than or equal to 18. [Page 92, Problem 40 (modified).]

9. Prove that if n is an integer , these four statements are equivalent: *(i) n is even, (ii) n+1 is odd, (iii) 3n+1 is odd, (iv) 3n is even.* [Page 92, Problem 41. Answer Page S-10.]

10. Prove that if n is an integer , these four statements are equivalent: *(i)* n^2 *is odd, (ii) 1-n is even, (iii)* n^3 *is odd, (iv)* $n^2 + 1$ *is even.* [Page 92, Problem 42.]