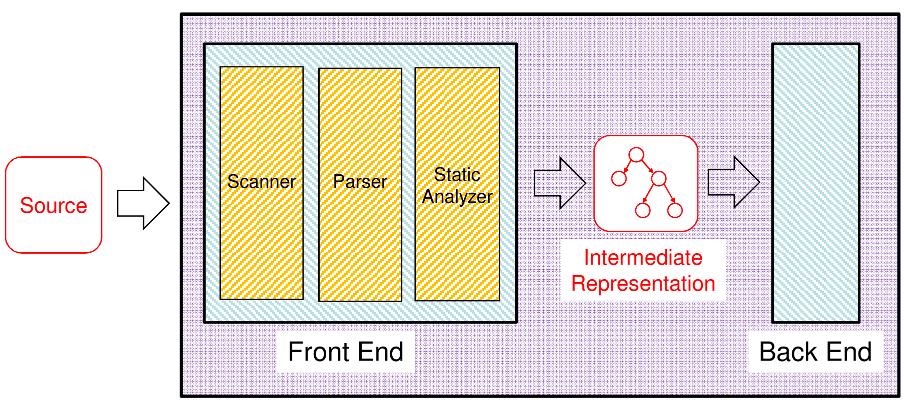
CMSC 330: Organization of Programming Languages

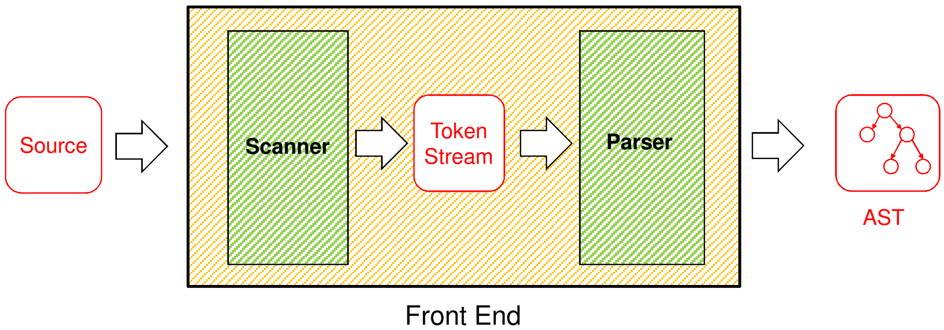
Context Free Grammars

Architecture of Compilers, Interpreters



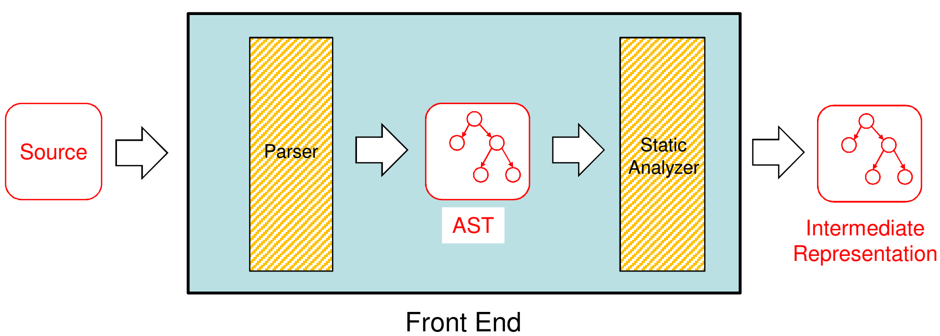
Compiler / Interpreter

Front End – Scanner



Scanner / lexer converts program source into tokens (keywords, variable names, operators, numbers, etc.) using regular expressions

Front End – Parser



Parser converts sequence of tokens into AST (abstract syntax tree) using context free grammars.

Context Free Grammar (CFG)

- A way of describing sets of strings (= languages)
  - The notation L(S) denotes the language of strings defined by S
- Example grammar: $S \rightarrow 0S \mid 1S \mid \varepsilon$

  String $s' \in L(S)$ iff
  - $s' = \varepsilon$, or $\exists s \in L(S)$ such that $s' = 0s$, or $s' = 1s$
- Grammar is same as regular expression (0|1)*
  - Generates / accepts the same set of strings

Context-Free Grammars (CFGs)

- But CFGs can do what regexps cannot
  - $S \rightarrow (S) \mid \varepsilon$ // represents balanced pairs of ( )'s
- In fact, CFGs subsume REs, DFAs, NFAs
  - There is a CFG that generates any regular language
  - But REs are a better notation for regular languages
- CFGs can specify programming language syntax
  - CFGs (mostly) describe the parsing process

Formal Definition: Context-Free Grammar

- A CFG G is a 4-tuple (Σ, N, P, S)
  - Σ – alphabet (finite set of symbols, or terminals)
    - Often written in lowercase
  - N – a finite, nonempty set of nonterminal symbols
    - Often written in uppercase
    - It must be that $N \cap \Sigma = \varnothing$
  - P – a set of productions of the form $N \to (\Sigma|N)^*$
    - Informally: the nonterminal can be replaced by the string of zero or more terminals / nonterminals to the right of the $\to$
    - Can think of productions as rewriting rules (more later)
  - $S \in N$ – the start symbol

Backus-Naur Form

- Context-free grammar production rules are also called Backus-Naur Form or BNF
  - A production like $A \to B\ c\ D$ is written in BNF as
    <A> ::= <B> c <D> (Non-terminals written with angle brackets and ::= instead of $\to$)
  - Often used to describe language syntax
- BNF was designed by
  - John Backus
    - Chair of the Algol committee in the early 1960s
  - Peter Naur
    - Secretary of the committee, who used this notation to describe Algol in 1962

Generating Strings

- We can think of a grammar as generating strings by rewriting
- Example grammar: $S \rightarrow 0S \mid 1S \mid \varepsilon$
  - $S \Rightarrow 0S$ // using $S \rightarrow 0S$
  - $\Rightarrow 01S$ // using $S \rightarrow 1S$
  - $\Rightarrow 011S$ // using $S \rightarrow 1S$
  - $\Rightarrow 011$ // using $S \rightarrow \varepsilon$

Accepting Strings (Informally)

- Determining if $s \in L(S)$ is called acceptance: goal is to find a rewriting from S that yields s
  - $011 \in L(S)$ according to the previous rewriting
  - A rewriting is some sequence of productions (rewrites) applied starting at the start symbol
- Terminology
  - Such a sequence of rewrites is a derivation or parse
  - Discovering the derivation is called parsing

Derivations

- Notation
  - $\Rightarrow$ indicates a derivation of one step
  - $\Rightarrow^+$ indicates a derivation of one or more steps
  - $\Rightarrow^*$ indicates a derivation of zero or more steps
- Example
  - $S \rightarrow 0S \mid 1S \mid \varepsilon$
- For the string 010
  - $S \Rightarrow 0S \Rightarrow 01S \Rightarrow 010S \Rightarrow 010$
  - $S \Rightarrow^+ 010$
  - $S \Rightarrow^* S$

Language Generated by Grammar

- L(G) the language defined by G is

$$L(G) = \{ \omega \in \Sigma^* \mid S \Rightarrow^+ \omega \}$$

  - S is the start symbol of the grammar
  - Σ is the alphabet for that grammar
- In other words
  - All strings over Σ that can be derived from the start symbol via one or more productions

Practice

- Try to make a grammar which accepts
  - $0^*|1^*$ – 0^n1^n where $n \geq 0$ – 0^n1^m where $m \leq n$

$S \rightarrow A \mid B$
$A \rightarrow 0A \mid \varepsilon$
$B \rightarrow 1B \mid \varepsilon$

$S \rightarrow 0S1 \mid \varepsilon$

$S \rightarrow 0S1 \mid 0S \mid \varepsilon$

- Give some example strings from this language
  - $S \rightarrow 0 \mid 1S$
    - 0, 10, 110, 1110, 11110, …
  - What language is it?
    - 1*0

Example: Arithmetic Expressions

- E → a | b | c | E+E | E-E | E*E | (E)
  - An expression E is either a letter a, b, or c
  - Or an E followed by + followed by an E
  - etc…
- This describes (or generates) a set of strings
  - {a, b, c, a+b, a+a, a*c, a-(b*a), c*(b + a), …}
- Example strings not in the language
  - d, c(a), a+, b**c, etc.

Formal Description of Example

- Formally, the grammar we just showed is
  - Σ = { +, -, *, (,), a, b, c }   // terminals
  - N = { E }   // nonterminals
  - P = { E → a, E → b, E → c,   // productions
    E → E-E, E → E+E,
    E → E*E,
    E → (E)
    }
  - S = E   // start symbol

(Non-)Uniqueness of Grammars

- Different grammars generate the same set of strings (language)
- The following grammar generates the same set of strings as the previous grammar

  E → E+T | E-T | T
  T → T*P | P
  P → (E) | a | b | c

Notational Shortcuts

- A production is of the form
  - left-hand side (LHS) $\rightarrow$ right hand side (RHS)
- If not specified
  - Assume LHS of first listed production is the start symbol
- Productions with the same LHS
  - Are usually combined with |
- If a production has an empty RHS
  - It means the RHS is ε

S → ABC // S is start symbol
A → aA
| b // A → b
| // A → ε

Practice

- Given the grammar

  S → aS | T
  T → bT | U
  U → cU | ε

  - Provide derivations for the following strings
    - b S ⇒ T ⇒ bT ⇒ bU ⇒ b
    - ac S ⇒ aS ⇒ aT ⇒ aU ⇒ acU ⇒ ac
    - bbc S ⇒ T ⇒ bT ⇒ bbT ⇒ bbU ⇒ bbcU ⇒ bbc
  - Does the grammar generate the following?
    - $S \Rightarrow^+$ ccc Yes $S \Rightarrow^+$ bS No
    - $S \Rightarrow^+$ bab No $S \Rightarrow^+$ Ta No

Practice

- Given the grammar

  $S \rightarrow aS \mid T$

  $T \rightarrow bT \mid U$

  $U \rightarrow cU \mid \varepsilon$

  - Name language accepted by grammar
    - a*b*c*
  - Give a different grammar accepting language

  $S \rightarrow ABC$
  $A \rightarrow aA \mid \varepsilon$ // a*
  $B \rightarrow bB \mid \varepsilon$ // b*
  $C \rightarrow cC \mid \varepsilon$ // c*

Parse Trees

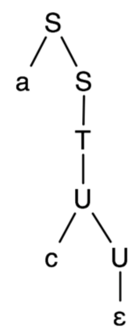
- Parse tree shows how a string is produced by a grammar
  - Root node is the start symbol
  - Every internal node is a nonterminal
  - Children of an internal node
    - Are symbols on RHS of production applied to nonterminal
  - Every leaf node is a terminal or ε

- Reading the leaves left to right
  - Shows the string corresponding to the tree

Parse Tree Example

S ⇒ aS ⇒ aT ⇒ aU ⇒ acU ⇒ ac

S → aS | T
T → bT | U
U → cU | ε



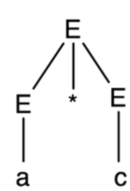
Parse Trees for Expressions

- A parse tree shows the structure of an expression as it corresponds to a grammar

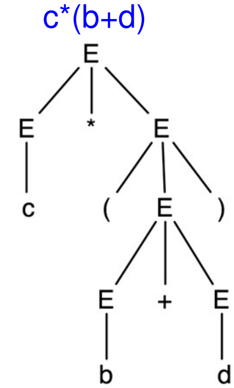
E → a | b | c | d | E+E | E-E | E*E | (E)

a

a*c



c*(b+d)



Practice

E → a | b | c | d | E+E | E-E | E*E | (E)

Make a parse tree for…

- a*b
- a+(b-c)
- d*(d+b)-a
- (a+b)*(c-d)
- a+(b-c)*d

Leftmost and Rightmost Derivation

- Leftmost derivation
  - Leftmost nonterminal is replaced in each step
- Rightmost derivation
  - Rightmost nonterminal is replaced in each step
- Example
  - Grammar
    - S → AB, A → a, B → b
  - Leftmost derivation for "ab"
    - S ⇒ AB ⇒ aB ⇒ ab
  - Rightmost derivation for "ab"
    - S ⇒ AB ⇒ Ab ⇒ ab

Parse Tree For Derivations

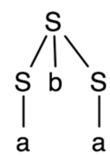
- Parse tree may be same for both leftmost & rightmost derivations
  - Example Grammar: $S \rightarrow a \mid SbS$ String: aba

    Leftmost Derivation

    $S \Rightarrow SbS \Rightarrow abS \Rightarrow aba$

    Rightmost Derivation

    $S \Rightarrow SbS \Rightarrow Sba \Rightarrow aba$



  - Parse trees don't show order productions are applied
- Every parse tree has a unique leftmost and a unique rightmost derivation

Parse Tree For Derivations (cont.)

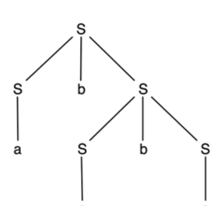
- Not every string has a unique parse tree
  - Example Grammar: $S \rightarrow a \mid SbS$ String: ababa

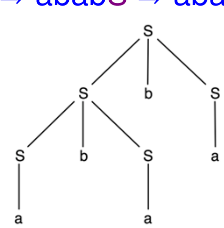
    Leftmost Derivation

    $S \Rightarrow SbS \Rightarrow abS \Rightarrow abSbS \Rightarrow ababS \Rightarrow ababa$

    Another Leftmost Derivation

    $S \Rightarrow SbS \Rightarrow SbSbS \Rightarrow abSbS \Rightarrow ababS \Rightarrow ababa$



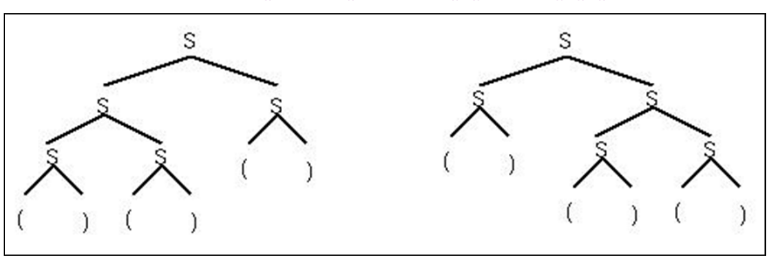


Ambiguity

- A grammar is ambiguous if a string may have multiple leftmost derivations
  - Equivalent to multiple parse trees
  - Can be hard to determine
    1. $S \rightarrow aS \mid T$
       $T \rightarrow bT \mid U$
       $U \rightarrow cU \mid \varepsilon$ **No**
    2. $S \rightarrow T \mid T$
       $T \rightarrow Tx \mid Tx \mid x \mid x$ **Yes**
    3. $S \rightarrow SS \mid () \mid (S)$ **?**

Ambiguity (cont.)

- Example
  - Grammar: $S \rightarrow SS \mid () \mid (S)$ String: ()()()
  - 2 distinct (leftmost) derivations (and parse trees)
    - $S \Rightarrow \underline{S}S \Rightarrow \underline{S}SS \Rightarrow ()\underline{S}S \Rightarrow ()()\underline{S} \Rightarrow ()()()$
    - $S \Rightarrow \underline{S}S \Rightarrow ()\underline{S} \Rightarrow ()\underline{S}S \Rightarrow ()()\underline{S} \Rightarrow ()()()$



CFGs for Programming Languages

- Recall that our goal is to describe programming languages with CFGs
- We had the following example which describes limited arithmetic expressions

  E → a | b | c | E+E | E-E | E*E | (E)

- What's wrong with using this grammar?
  - It's ambiguous!

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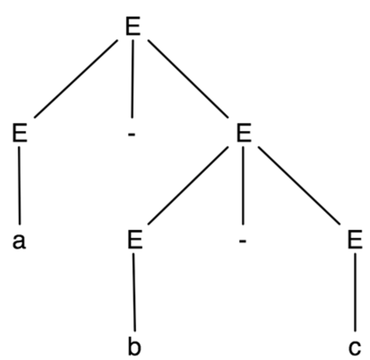
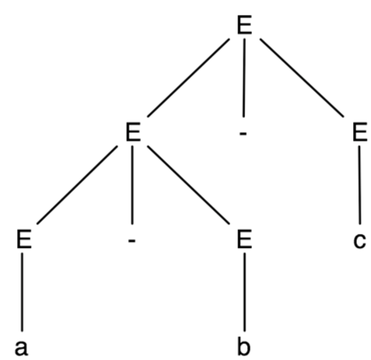
Example: a-b-c

E ⇒ E-E ⇒ a-E ⇒ a-E-E ⇒ a-b-E ⇒ a-b-c



Corresponds to a-(b-c)

E ⇒ E-E ⇒ E-E-E ⇒ a-E-E ⇒ a-b-E ⇒ a-b-c



Corresponds to (a-b)-c

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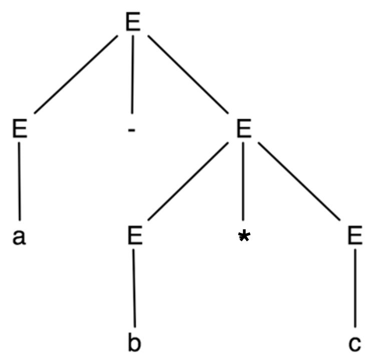
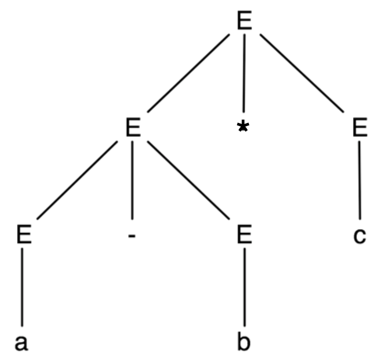
Example: a-b*c

E ⇒ E-E ⇒ a-E ⇒ a-E*E ⇒
a-b*E ⇒ a-b*c



Corresponds to a-(b*c)

E ⇒ E-E ⇒ E-E*E ⇒
a-E*E ⇒ a-b*E ⇒ a-b*c



Corresponds to (a-b)*c

Another Example: If-Then-Else

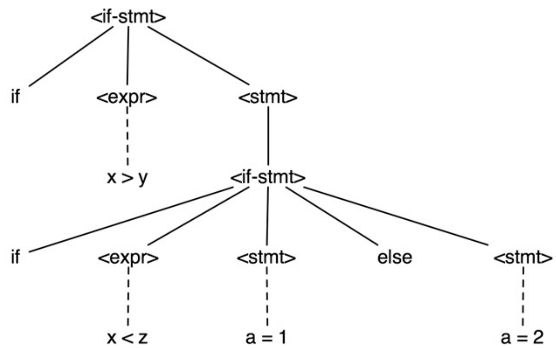
<stmt> → <assignment> | <if-stmt> | ...
<if-stmt> → if (<expr>) <stmt> |
if (<expr>) <stmt> else <stmt>
(Note < >'s are used to denote nonterminals)

- Consider the following program fragment

```
if (x > y)
  if (x < z)
    a = 1;
  else a = 2;
```

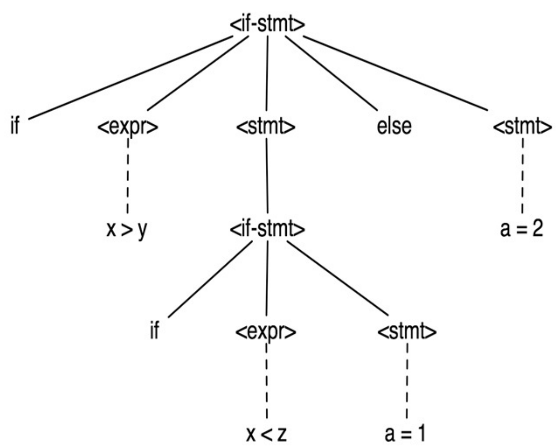
(Note: Ignore newlines)

Parse Tree #1



- Else belongs to inner if

Parse Tree #2



- Else belongs to outer if

Dealing With Ambiguous Grammars

- Ambiguity is bad
  - Syntax is correct
  - But semantics differ depending on choice
    - Different associativity (a-b)-c vs. a-(b-c)
    - Different precedence (a-b)*c vs. a-(b*c)
    - Different control flow if (if else) vs. if (if) else
- Two approaches
  - Rewrite grammar
  - Use special parsing rules
    - Depending on parsing method (learn in CMSC 430)

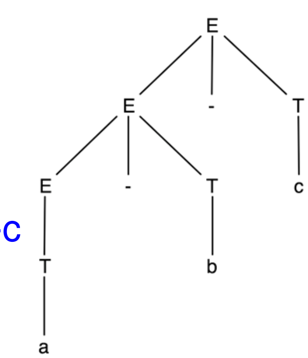
Fixing the Expression Grammar

- Require right operand to not be bare expression

  $E \rightarrow E+T \mid E-T \mid E*T \mid T$

  $T \rightarrow a \mid b \mid c \mid (E)$

- Corresponds to left associativity
- Now only one parse tree for a-b-c
  - Find derivation



What If We Want Right Associativity?

- Left-recursive productions
  - Used for left-associative operators
  - Example

    $E \rightarrow E+T \mid E-T \mid E*T \mid T$

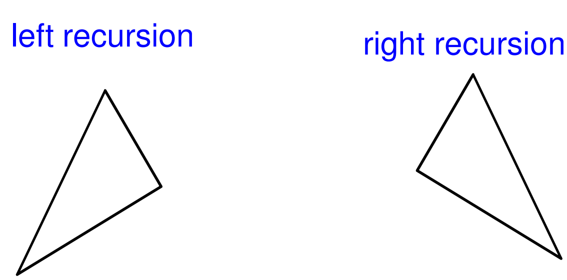
    $T \rightarrow a \mid b \mid c \mid (E)$
- Right-recursive productions
  - Used for right-associative operators
  - Example

    $E \rightarrow T+E \mid T-E \mid T*E \mid T$

    $T \rightarrow a \mid b \mid c \mid (E)$

Parse Tree Shape

- The kind of recursion determines the shape of the parse tree

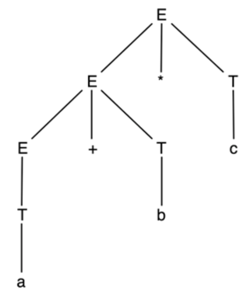


A Different Problem

- How about the string a+b*c ?

  E → E+T | E-T | E*T | T

  T → a | b | c | (E)



- Doesn't have correct precedence for *
  - When a nonterminal has productions for several operators, they effectively have the same precedence
- Solution – Introduce new nonterminals

Final Expression Grammar

E → E+T | E-T | T	lowest precedence operators
T → T*P | P	higher precedence
P → a | b | c | (E)	highest precedence (parentheses)

- Controlling precedence of operators
  - Introduce new nonterminals
  - Precedence increases closer to operands
- Controlling associativity of operators
  - Introduce new nonterminals
  - Assign associativity based on production form
    - E → E+T (left associative) vs. E → T+E (right associative)

Tips For Designing Grammars

1. Use recursive productions to generate an arbitrary number of symbols

   $A \rightarrow xA \mid \varepsilon$ // Zero or more x's

   $A \rightarrow yA \mid y$ // One or more y's

2. Use separate non-terminals to generate disjoint parts of a language, and then combine in a production

   { a*b* } // a's followed by b's

   $S \rightarrow AB$

   $A \rightarrow aA \mid \varepsilon$ // Zero or more a's

   $B \rightarrow bB \mid \varepsilon$ // Zero or more b's

Tips For Designing Grammars (cont.)

3. To generate languages with matching, balanced, or related numbers of symbols, write productions which generate strings from the middle

   $\{a^n b^n \mid n \geq 0\}$ // N a's followed by N b's

   $S \rightarrow aSb \mid \varepsilon$

   Example derivation: $S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$

   $\{a^n b^{2n} \mid n \geq 0\}$ // N a's followed by 2N b's

   $S \rightarrow aSbb \mid \varepsilon$

   Example derivation: $S \Rightarrow aSbb \Rightarrow aaSbbbb \Rightarrow aabbbb$

Tips For Designing Grammars (cont.)

4. For a language that is the union of other languages, use separate nonterminals for each part of the union and then combine

$\{ a^n(b^m|c^m) \mid m > n \geq 0\}$

Can be rewritten as

$\{ a^nb^m \mid m > n \geq 0\} \cup \{ a^nc^m \mid m > n \geq 0\}$

$S \rightarrow T \mid V$
$T \rightarrow aTb \mid U$
$U \rightarrow Ub \mid b$
$V \rightarrow aVc \mid W$
$W \rightarrow Wc \mid c$