

Problem 1: Palindromes

Let $S = (s_1 s_2 \dots s_n)$ be the input sequence. Define $A[i, j]$ as the number of ways one can remove a few symbols (maybe $\emptyset$) from sequence $(s_i \dots s_j)$ such that the rest of sequence $(s_i \dots s_j)$ becomes a palindrome. Write a dynamic program to compute $A[i, j]$ based on $A[i + 1, j]$, $A[i, j - 1]$, and $A[i + 1, j - 1]$.

Problem 2: Tree

For each vertex v , compute the final answer to query “find v ” and store it in $result[v]$. You can fill array $result$ by one DFS.

- For each query “find v ”, you can output $result[v]$.
- For each query “change v w ”, you should update $result[u]$ for each vertex u in the subtree rooted at v (including v). Design an algorithm for updating array $result$ by one DFS from vertex v . Note that there are at most 100 queries of this type.

Problem 3: Increasing Shortest Path

Sort all edges regarding their weights and let $(e_1, e_2, \dots, e_m)$ be the result, i.e., $w_{e_1} \leq w_{e_2} \leq \dots \leq w_{e_m}$. Now, consider a query which is represented by A and B. This means you need to find the shortest path (the path with minimum sum of weights of its edges) which goes from node A to node B such that the weights of the edges in that path are in increasing order along the path.

Let $B[i]$ be the minimum sum of weights for a path starting at node A and ending at edge e_i which satisfies the given constraints. Write a dynamic program for computing $B[1], B[2], \dots, B[m]$.

Given array B, what would be the final answer?