

CMSC330 Practice Problems 8

1. Lambda calculus

Make all parentheses explicit in the following λ -expressions

- a. $\lambda x.xz \lambda y.xy$
- b. $(\lambda x.xz) \lambda y.w \lambda w.wyzx$
- c. $\lambda x.xy \lambda x.yx$

Find all free (unbound) variables in the following λ -expressions

- d. $\lambda x.x z \lambda y.x y$
- e. $(\lambda x. x z) \lambda y. w \lambda w. w y z x$
- f. $\lambda x. x y \lambda x. y x$

Apply β -reduction to the following λ -expressions as much as possible

- g. $(\lambda z.z) (\lambda y.y y) (\lambda x.x a)$
- h. $(\lambda z.z) (\lambda z.z z) (\lambda z.z y)$
- i. $(\lambda x.\lambda y.x y y) (\lambda a.a) b$
- j. $(\lambda x.\lambda y.x y y) (\lambda y.y) y$
- k. $(\lambda x.x x) (\lambda y.y x) z$
- l. $(\lambda x. (\lambda y. (x y)) y) z$
- m. $((\lambda x.x x) (\lambda y.y)) (\lambda y.y)$
- n. $((\lambda x. \lambda y.(x y))(\lambda y.y)) w$

Show that the following expression has multiple reduction sequences

- o. $(\lambda x.y) ((\lambda y.y y y) (\lambda x.x x x))$

2. Lambda calculus encodings

Prove the following using the appropriate λ -calculus encodings

- a. $\text{not} (\text{not true}) = \text{true}$
- b. $\text{or false true} = \text{true}$
- c. $\text{if false then } x \text{ else } y = y$
- d. $\text{succ } 2 = 3$
- e. $(* 1 3) = 3$
- f. $(+ 2 1) = 3$
- g. $(Y \text{ fact}) 2 = 2$ // you do not need to expand any operators except fact & Y