CMSC330 Spring 2010 Midterm #2 Solutions

1. (16 pts) OCaml Types and Type Inference

   Give the type of the following OCaml expression

   a. (2 pts) [[1 ; 2]] **Type = int list list**

   b. (3 pts) fun x -> 2::x **Type = (int list) -> (int list)**

   Write an OCaml expression with the following type

   c. (2 pts) int list -> int **Code = fun (x::_) -> x+2**

   d. (4 pts) (int -> bool) -> int **Code = fun x -> if (x 1) then 2 else 3**

   Give the value of the following OCaml expression. If an error exists, describe it

   e. (2 pts) if (1 < 2) then 3 **Value = error since 3 must be of type ( )**

   f. (3 pts) let f x = f 2 in 1 **Value = error since f is undefined**

2. (14 pts) Higher order & anonymous functions

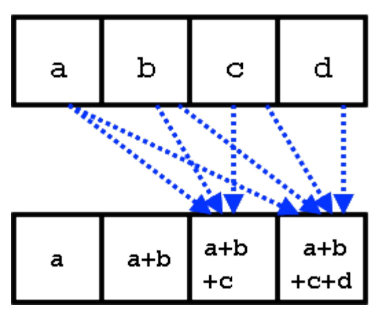
   A *prefix sum* is an operation on lists in which the n^{th} element in the result list is obtained from the sum of the first *n* elements in the operand list. Using the following code for fold and an anonymous function, write a function prefixSum which given a list of ints, returns the prefix sum for the list.

   You are not allowed to use any helper functions or OCaml library functions, with the exception of List.rev (which reverses a list).

   Partial credit given for solutions which do not use fold.

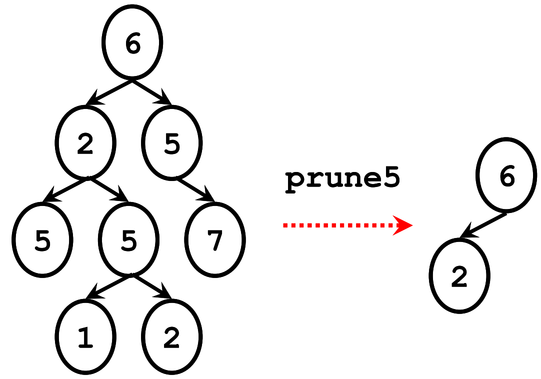
   Example: prefixSum [ ] = [ ]
   prefixSum [1;1;1;1;1] = [1;2;3;4;5]
   prefixSum [1;2;3;4] = [1;3;6;10]

```
let rec fold f a lst = match lst with
   [] -> a
   | (h::t) -> fold f (f a h) t
```




```
let prefixSum lst = List.rev (fold
      (fun a y -> match a with
                  [ ] -> [y]
                  | (h::_) -> ((h+y)::a))
      [ ] lst) ;;
```

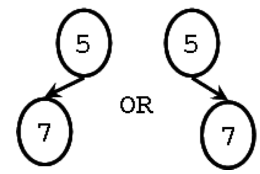
3. (16 pts) OCaml polymorphic datatypes



Consider the OCaml type *tree* implementing a binary tree of ints:

```
type tree =
   Empty
   | Node of int * tree * tree;;
```

a. (4 pts) Write an OCaml expression creating the data structure for a binary tree where the root node has value 5 and has one child node with value 7.



Node (5, Empty, Node (7, Empty, Empty)) **OR**
Node (5, Node (7, Empty, Empty), Empty)

b. (5 pts) Implement a function *count5* that takes a tree and returns the number of nodes with the value 5. You may use helper functions (though they are not needed).

```
let rec count5 = function
   Empty -> 0                                         // 0 if empty
 | Node (n, lt, rt) ->                                // examine node
    (if (n=5) then 1 else 0) + (count5 lt) + (count5 rt)   // recurse on subtrees
```

c. (7 pts) Implement a function *prune5* that takes a tree and returns a tree where all nodes with the value 5 (and their subtrees) are removed. You may use helper functions (though they are not needed).

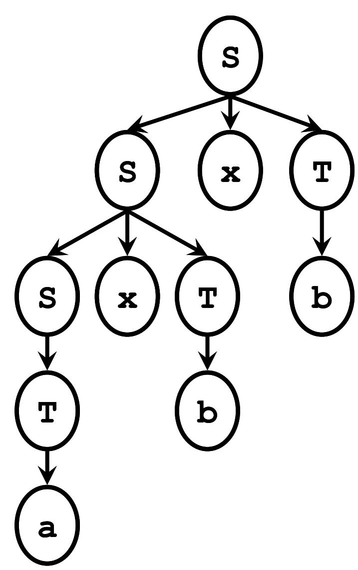
```
let rec prune5 = function
   Empty -> Empty                              // no change if empty
 | Node (n, lt, rt) ->                         // examine node
    if (n=5) then Empty                        // if 5 then prune
    else (Node (n, prune5 lt, prune5 rt))      // recurse on subtrees
```

4. (16 pts) Context free grammars
Consider the following grammar: S → S x T | T   T → a | b

a. (3 pts) Describe the set of strings generated by the grammar
((a|b)x)*(a|b) **OR** **(a|b)(x(a|b))***

b. (3 pts) Provide a left-most derivation for the string "axbxb".
S ⇒ SxT ⇒ SxTxT ⇒ TxTxT ⇒ axTxT ⇒ axbxT ⇒ axbxb

c. (2 pts) Provide a parse tree for the string "axbxb".
See right



d. (2 pts) What is the associativity of the x operator for the grammar?

    Left associative

e. (6 pts) Apply the algorithm discussed in class to transform the grammar so that it can be parsed using a recursive descent parser.

    S → T L
    L → x T L | ε
    T → a | b

5. (22 pts) Parsing

Consider the following grammar

S → Abc | dS | ε (* epsilon *)
A → aSA | f

a. (8 pts) Compute First sets for S and A

    First(S) = { a, d, f, ε }
    First(A) = { a, f }

b. (14 pts) Using pseudocode, write a recursive descent parser for the grammar. Use the following utilities:

lookahead	Variable holding next terminal Lookahead == "$" when at end of input
match (x)	Function to match next terminal to x
error ( )	Reports parse error for input

```
parse_S( ) {
        if (lookahead == "a") || (lookahead == "f") {        // S → Abc
                parse_A( ); match("b"); match("c") ;
        }
        else if (lookahead == "d") {                         // S → dS
                match( "d" ); parse_S( ) ;
        }
        else                                                 // S → ε
                ;
    }

parse_A( ) {
        if (lookahead == "a") {                              // A → aSA
                match( "a" ); parse_S( ) ; parse_A( );
        }
        else if (lookahead == "f") {                         // A → f
                match( "f" );
        }
        else
                error( );
}
```

6. (16 pts) Operational semantics

   a. (4 pts) Consider the following operational semantics judgement. State in English what this statement is expressing:

      $\bullet , x{:}1 ; (+\ x\ 2) \to 3$

      The expression (+ x 2) evaluates to the value 3 in the environment resulting from the empty environment adding the binding x=1.

   b. (12 pts) In an empty environment, to what value v will the expression

      (fun z = z) (+ 1 2)

      evaluate to? In other words, find a v such that you can prove the following:

      $\bullet\ ; (\text{fun } z = z)\ (+\ 1\ 2) \to v$

      Use the operational semantics rules given in class. Show the complete proof that stacks uses of these rules.

$$\frac{\underline{\bullet ; (\text{fun } z = z) \to (\bullet , \lambda z.z)} \qquad \dfrac{\bullet ; 1 \to 1 \qquad \bullet ; 2 \to 2}{\bullet ; (+\ 1\ 2) \to 3} \qquad (z{:}3 ; z) \to 3}{\bullet ; (\text{fun } z = z)\ (+\ 1\ 2) \to 3}$$