1. A *simple cycle* is a cycle in a graph that does not repeat any vertices (except the start and finish vertex). The optimization version of the *Longest Simple Cycle Problem* is to find a simple cycle with as many edges as possible.

   (a) Define the decision version of the Longest Simple Cycle Problem.

   (b) Show that the decision version of the Longest Simple Cycle Problem for undirected graphs is in **NP**. What is the certificate?

   (c) Show that, if you can solve the optimization version of the Longest Simple Cycle Problem for undirected graphs in polynomial time, then you can also solve the decision version in polynomial time.

   (d) Show that, if you can solve the decision version of the Longest Simple Cycle Problem for undirected graphs in polynomial time then, you can also solve the optimization version in polynomial time. HINT: First determine the size of the longest simple cycle.

2. A *complete graph* is a graph with every possible edge. So the adjacency matrix for a complete, **directed** graph has all 1's, and the adjacency matrix for a complete, **undirected** graph has all 1's except the main diagonal has all 0's.

   We would like to partition the edges of a graph into Hamiltonian cycles (with possibly some edges left over).

   The optimization version of the *Hamiltonian Cycle Partition Problem* is to partition the edges of a graph into as many Hamiltonian Cycles as possible (with possibly some edges left over).

   (a) Solve the optimization version of the Hamiltonian Cycle Partition Problem for complete graphs. Your answer should be a function of the number of vertices n. Justify your answers.

      i. For complete, **directed** graphs. (For $n = 0$ there are no Hamiltonian cycles. for $n = 1, 2$ there is one Hamiltonian cycle. For $n = 3$ there are two Hamiltonian cycles.)

      ii. For complete, **undirected** graphs. (For $n = 0, 1, 2$ there are no Hamiltonian cycles. For $n = 3$ there is one Hamiltonian cycle, which is the graph itself.)

   (b) Define the decision version of the *Hamiltonian Cycle Partition Problem* for undirected graphs.

   (c) Show that the decision version of the Hamiltonian Cycle Partition Problem for undirected graphs is in **NP**.

   (d) Show that if you can solve the optimization version of the Hamiltonian Cycle Partition Problem in polynomial time for undirected graphs then you can also solve the decision version in polynomial time.

   (e) Show that if you can solve the decision version of the Hamiltonian Cycle Partition Problem for undirected graphs in polynomial time then you can also solve the optimization version in polynomial time. HINT: First determine how many cycles are in the partition and then find the cycles.