There are five questions.

1. (a) Assume you have an alphabet of letters from "a" though "g" plus "s" and "t". Illustrate the operation of radix sort on the following list of English words:

   sass, baes, ebbs, test, tats, tete, cast, acts, tact, sate

   (b) Use "bae" and "tete-a-tete" in an English sentence that shows that you understand the meaning of both.

2. Professor Aristotle tells you excitedly that he has an improvement on Counting Sort that sorts n integers in the range $0, \ldots, k-1$ in time $\Theta(n + \sqrt{k})$. He then asks you what improvement you can get on Radix Sort using his wonderful new algorithm.

   (a) Let n be the number of elements to be sorted, r the radix, and d the number of digits. Using Professor Aristotle's version of Counting Sort, what is the time for Radix Sort in terms of n, r, and d?

   (b) Assume all of the elements are integers in the range $0, \ldots, S-1$, what is the relationship between r, d, and S?

   (c) Rewrite the running time of the new Radix Sort in terms of n, r, and S.

   (d) Find the high order term for optimal value for r (in terms of n and S):

      i. Drop the order notation and take the derivative of your formula with respect to r.

      ii. Set the numerator equal to 0 and solve for r dropping low order terms. Your solution should have r by itself on the left side of the equality and $\ln r$ on the right side, which is an implicit equation for r.

      iii. Guess that $r = e$. Substitute into the right side and simplify dropping low order terms. Using this as your new guess for r, substitute into the right side and simplify dropping low order terms. Keep doing this until you get the same value back for r. This will be the high order term for the optimal value of r.

   (e) Rewrite the running time of the new Radix Sort in terms of n and S, using your optimal value of r.

   (f) What should you tell Professor Aristotle? Your answer should be brief, useful, cogent, and cover the important points!

   (g) BTW. Do you believe that Professor Aristotle has actually found such an algorithm? Why or why not?

3. Let A be a $p \times q$ matrix and B be a $q \times r$ matrix. Then their product $C = AB$ is a $p \times r$ matrix, where
$$c_{i,j} = \sum_{k=1}^{q} a_{ik} b_{kj} \ .$$
Multiply by hand
$$\begin{bmatrix} 3 & -2 & -3 \\ 1 & 4 & 5 \end{bmatrix} \begin{bmatrix} 6 & -3 & 0 & 4 \\ -5 & 0 & 5 & 3 \\ 1 & 0 & -2 & 2 \end{bmatrix}$$

4. Assume you multiply a $p \times q$ matrix with a $q \times r$ matrix.
   (a) Exactly how many atomic multiplies does it take? Justify briefly.
   (b) Exactly how many atomic additions does it take? Justify briefly.

5. Assume you multiply two $n \times n$ matrices.
   (a) Exactly how many atomic multiplies does it take?
   (b) Exactly how many atomic additions does it take?