Parallel Databases; Map-Reduce

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Client-Server Systems

Database functionality can be divided into:

- **Back-end**: manages access structures, query evaluation and optimization, concurrency control and recovery.
- **Front-end**: consists of tools such as *forms*, *report-writers*, and graphical user interface facilities.

The interface between the front-end and the back-end is through SQL or through an application program interface.
Parallel Databases

- Why?
  - More transactions per second, or less time per query
  - Throughput vs. Response Time
  - Speedup vs. Scaleup

- Database operations are *embarrassingly parallel*
  - E.g. Consider a join between R and S on R.b = S.b

- But, perfect speedup doesn’t happen
  - Start-up costs
  - Interference
  - Skew
Parallel Systems

- Parallel database systems consist of multiple processors and multiple disks connected by a fast interconnection network.

- A **coarse-grain parallel** machine consists of a small number of powerful processors.

- A **massively parallel** or **fine grain parallel** machine utilizes thousands of smaller processors.

- Two main performance measures:
  - **throughput** --- the number of tasks that can be completed in a given time interval
  - **response time** --- the amount of time it takes to complete a single task from the time it is submitted
Speed-Up and Scale-Up

- **Speedup**: a fixed-sized problem executing on a small system is given to a system which is \( N \)-times larger.
  - Measured by:
    \[
    \text{speedup} = \frac{\text{small system elapsed time}}{\text{large system elapsed time}}
    \]
  - Speedup is **linear** if equation equals \( N \).

- **Scaleup**: increase the size of both the problem and the system
  - \( N \)-times larger system used to perform \( N \)-times larger job
  - Measured by:
    \[
    \text{scaleup} = \frac{\text{small system small problem elapsed time}}{\text{big system big problem elapsed time}}
    \]
  - Scale up is **linear** if equation equals 1.
Speedup

- Linear speedup
- Sublinear speedup

(speed vs. resources)
Scaleup

- **linear scaleup**
- **sublinear scaleup**

\[
\frac{T_S}{T_L}
\]
Factors Limiting Speedup and Scaleup

Speedup and scaleup are often sublinear due to:

- **Startup costs**: Cost of starting up multiple processes may dominate computation time, if the degree of parallelism is high.

- **Interference**: Processes accessing shared resources (e.g., system bus, disks, or locks) compete with each other, thus spending time waiting on other processes, rather than performing useful work.

- **Skew**: Increasing the degree of parallelism increases the variance in service times of parallelly executing tasks. Overall execution time determined by **slowest** of parallelly executing tasks.
Parallel Databases

- Shared-nothing vs. shared-memory vs. shared-disk

(a) shared memory

(b) shared disk

(c) shared nothing

(d) hierarchical
# Parallel Databases

<table>
<thead>
<tr>
<th></th>
<th>Shared Memory</th>
<th>Shared Disk</th>
<th>Shared Nothing</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Communication</strong></td>
<td>Extremely fast</td>
<td>Disk interconnect is very fast</td>
<td>Over a LAN, so slowest</td>
</tr>
<tr>
<td>between processors</td>
<td></td>
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<tr>
<td><strong>Scalability ?</strong></td>
<td>Not beyond 32 or 64 or so (memory bus is the bottleneck)</td>
<td>Not very scalable (disk interconnect is the bottleneck)</td>
<td>Very very scalable</td>
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<tr>
<td><strong>Notes</strong></td>
<td>Cache-coherency an issue</td>
<td>Transactions complicated; natural fault-tolerance.</td>
<td>Distributed transactions are complicated (deadlock detection etc);</td>
</tr>
<tr>
<td><strong>Main use</strong></td>
<td>Low degrees of parallelism</td>
<td>Not used very often</td>
<td>Everywhere</td>
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Distributed Systems

- Over a wide area network
  - Typically not done for *performance reasons*
    - For that, use a parallel system
  - Done because of necessity
    - Imagine a large corporation with offices all over the world
    - Also, for redundancy and for disaster recovery reasons
  - Lot of headaches
    - Especially if trying to execute transactions that involve data from multiple sites
      - Keeping the databases in sync
        - *2-phase commit* for transactions uniformly hated
      - Autonomy issues
        - Even within an organization, people tend to be protective of their unit/department
      - Locks/Deadlock management
  - Works better for query processing
    - Since we are only reading the data
MapReduce Framework

- Provides a fairly restricted, but still powerful abstraction for programming

- Programmers write a pipeline of functions, called map or reduce
  - map programs
    - inputs: a list of “records” (record defined arbitrarily – could be images, genomes etc…)
    - output: for each record, produce a set of “(key, value)” pairs
  - reduce programs
    - input: a list of “(key, {values})” grouped together from the mapper
    - output: whatever

- Both can do arbitrary computations on the input data as long as the basic structure is followed
MapReduce Framework

- input files
- mappers
- intermediate files
- reducers
- output files
Word Count Example

```java
map(String key, String value):
    // key: document name
    // value: document contents
    for each word w in value:
        EmitIntermediate(w, "1");

reduce(String key, Iterator values):
    // key: a word
    // values: a list of counts
    int result = 0;
    for each v in values:
        result += ParseInt(v);
    Emit(AsString(result));
```
MapReduce Framework: Word Count

**input files**
- a b a c d b
- b c d a a a
- a b a b a b
- c c c c c

**mappers**
- (a, 1)
- (b, 1)
- (a, 1)
- (c, 1)
- (d, 1)
- (b, 1)

**intermediate files**
- (a, 1)
- (a, 1)
- (c, 1)
- (a, 1)
- (a, 1)
- ...

**reducers**
- (b, 1)
- (d, 1)
- (b, 1)
- (b, 1)
- (d, 1)
- (b, 1)
- ...

**output files**
- (a, 8)
- (c, 5)
- (b, 6)
- (d, 2)
More Efficient Word Count

**input files**
- a b a c d b
- b c d a a a
- a b a b a b
- c c c c c

**mappers**
- (a, 2)
- (b, 2)
- (c, 1)
- (d, 1)

**intermediate files**
- (a, 2)
- (a, 3)
- (c, 1)
- (c, 5)

**reducers**
- (a, 8)
- (c, 5)

**output files**
- (b, 6)
- (d, 2)

Called “mapper-side” combiner
Chapter 18: Parallel Databases

- Introduction
- I/O Parallelism
- Interquery Parallelism
- Intraquery Parallelism
- Intraoperation Parallelism
- Interoperation Parallelism
- Design of Parallel Systems
Introduction

- Parallel machines are becoming quite common and affordable
  - Prices of microprocessors, memory and disks have dropped sharply
  - Recent desktop computers feature multiple processors and this trend is projected to accelerate

- Databases are growing increasingly large
  - Large volumes of transaction data are collected and stored for later analysis.
  - Multimedia objects like images are increasingly stored in databases

- Large-scale parallel database systems increasingly used for:
  - Storing large volumes of data
  - Processing time-consuming decision-support queries
  - Providing high throughput for transaction processing
Parallelism in Databases

- Data can be partitioned across multiple disks for parallel I/O.
- Individual relational operations (e.g., sort, join, aggregation) can be executed in parallel
  - data can be partitioned and each processor can work independently on its own partition.
- Queries are expressed in high level language (SQL, translated to relational algebra)
  - makes parallelization easier.
- Different queries can be run in parallel with each other. Concurrency control takes care of conflicts.
- Thus, databases naturally lend themselves to parallelism.
I/O Parallelism

- Reduce the time required to retrieve relations from disk by partitioning.
- The relations on multiple disks.
- Horizontal partitioning – tuples of a relation are divided among many disks such that each tuple resides on one disk.
- Partitioning techniques (number of disks = $n$):
  - **Round-robin:**
    
    Send the $l$th tuple inserted in the relation to disk $i \mod n$.
  - **Hash partitioning:**
    
    - Choose one or more attributes as the partitioning attributes.
    - Choose hash function $h$ with range $0 \ldots n - 1$
    - Let $i$ denote result of hash function $h$ applied to the partitioning attribute value of a tuple. Send tuple to disk $i$. 
Partitioning techniques (cont.):

**Range partitioning:**

- Choose an attribute as the partitioning attribute.
- A partitioning vector \([v_0, v_1, ..., v_{n-2}]\) is chosen.
- Let \(v\) be the partitioning attribute value of a tuple. Tuples such that \(v_i \leq v_{i+1}\) go to disk \(I + 1\). Tuples with \(v < v_0\) go to disk 0 and tuples with \(v \geq v_{n-2}\) go to disk \(n-1\).

E.g., with a partitioning vector \([5,11]\), a tuple with partitioning attribute value of 2 will go to disk 0, a tuple with value 8 will go to disk 1, while a tuple with value 20 will go to disk 2.
Comparison of Partitioning Techniques

Evaluate how well partitioning techniques support the following types of data access:

1. Scanning the entire relation.
2. Locating a tuple associatively – **point queries**.
   - E.g., \( r.A = 25 \).
3. Locating all tuples such that the value of a given attribute lies within a specified range – **range queries**.
   - E.g., \( 10 \leq r.A < 25 \).
Comparison of Partitioning Techniques (Cont.)

Round robin:
- **Advantages**
  - Best suited for sequential scan of entire relation on each query.
  - All disks have almost an equal number of tuples; retrieval work is thus well balanced between disks.
- **Range queries are difficult to process**
  - No clustering -- tuples are scattered across all disks
Comparison of Partitioning Techniques (Cont.)

Hash partitioning:

- Good for sequential access
  - Assuming hash function is good, and partitioning attributes form a key, tuples will be equally distributed between disks
  - Retrieval work is then well balanced between disks.
- Good for point queries on partitioning attribute
  - Can lookup single disk, leaving others available for answering other queries.
  - Index on partitioning attribute can be local to disk, making lookup and update more efficient
- No clustering, so difficult to answer range queries
Comparison of Partitioning Techniques (Cont.)

- Range partitioning:
- Provides data clustering by partitioning attribute value.
- Good for sequential access
- Good for point queries on partitioning attribute: only one disk needs to be accessed.
- For range queries on partitioning attribute, one to a few disks may need to be accessed
  - Remaining disks are available for other queries.
  - Good if result tuples are from one to a few blocks.
  - If many blocks are to be fetched, they are still fetched from one to a few disks, and potential parallelism in disk access is wasted
    - Example of execution skew.
Partitioning a Relation across Disks

- If a relation contains only a few tuples which will fit into a single disk block, then assign the relation to a single disk.
- Large relations are preferably partitioned across all the available disks.
- If a relation consists of $m$ disk blocks and there are $n$ disks available in the system, then the relation should be allocated $\min(m,n)$ disks.
Handling of Skew

- The distribution of tuples to disks may be skewed — that is, some disks have many tuples, while others may have fewer tuples.

- Types of skew:
  - Attribute-value skew.
    - Some values appear in the partitioning attributes of many tuples; all the tuples with the same value for the partitioning attribute end up in the same partition.
    - Can occur with range-partitioning and hash-partitioning.
  - Partition skew.
    - With range-partitioning, badly chosen partition vector may assign too many tuples to some partitions and too few to others.
    - Less likely with hash-partitioning if a good hash-function is chosen.
Handling Skew in Range-Partitioning

- To create a **balanced partitioning vector** (assuming partitioning attribute forms a key of the relation):
  - Sort the relation on the partitioning attribute.
  - Construct the partition vector by scanning the relation in sorted order as follows.
    - After every $1/n^{th}$ of the relation has been read, the value of the partitioning attribute of the next tuple is added to the partition vector.
  - $n$ denotes the number of partitions to be constructed.
  - Duplicate entries or imbalances can result if duplicates are present in partitioning attributes.

- Alternative technique based on **histograms** used in practice
Handling Skew using Histograms

- Balanced partitioning vector can be constructed from histogram in a relatively straightforward fashion
  - Assume uniform distribution within each range of the histogram
- Histogram can be constructed by scanning relation, or sampling (blocks containing) tuples of the relation
Handling Skew Using Virtual Processor Partitioning

Skew in range partitioning can be handled elegantly using virtual processor partitioning:

- create a large number of partitions (say 10 to 20 times the number of processors)
- Assign virtual processors to partitions either in round-robin fashion or based on estimated cost of processing each virtual partition

Basic idea:

- If any normal partition would have been skewed, it is very likely the skew is spread over a number of virtual partitions
- Skewed virtual partitions get spread across a number of processors, so work gets distributed evenly!
Interquery Parallelism

- Queries/transactions execute in parallel with one another.
- Increases transaction throughput; used primarily to scale up a transaction processing system to support a larger number of transactions per second.
- Easiest form of parallelism to support, particularly in a shared-memory parallel database, because even sequential database systems support concurrent processing.
- More complicated to implement on shared-disk or shared-nothing architectures
  - Locking and logging must be coordinated by passing messages between processors.
  - Data in a local buffer may have been updated at another processor.
  - **Cache-coherency** has to be maintained — reads and writes of data in buffer must find latest version of data.
Example of a cache coherency protocol for shared disk systems:

- Before reading/writing to a page, the page must be locked in shared/exclusive mode.
- On locking a page, the page must be read from disk.
- Before unlocking a page, the page must be written to disk if it was modified.

More complex protocols with fewer disk reads/writes exist.

Cache coherency protocols for shared-nothing systems are similar. Each database page is assigned a home processor. Requests to fetch the page or write it to disk are sent to the home processor.
Intraquery Parallelism

- Execution of a single query in parallel on multiple processors/disks; important for speeding up long-running queries.

- Two complementary forms of intraquery parallelism:
  - **Intraoperation Parallelism** – parallelize the execution of each individual operation in the query.
  - **Interoperation Parallelism** – execute the different operations in a query expression in parallel.

  The first form scales better with increasing parallelism because the number of tuples processed by each operation is typically more than the number of operations in a query.
Our discussion of parallel algorithms assumes:

- *read-only* queries
- shared-nothing architecture
- *n* processors, $P_0, \ldots, P_{n-1}$, and *n* disks $D_0, \ldots, D_{n-1}$, where disk $D_i$ is associated with processor $P_i$.

If a processor has multiple disks they can simply simulate a single disk $D_i$.

Shared-nothing architectures can be efficiently simulated on shared-memory and shared-disk systems.

- Algorithms for shared-nothing systems can thus be run on shared-memory and shared-disk systems.
- However, some optimizations may be possible.
Parallel Sort

Range-Partitioning Sort

- Choose processors $P_0, ..., P_m$, where $m \leq n - 1$ to do sorting.
- Create range-partition vector with $m$ entries, on the sorting attributes
- Redistribute the relation using range partitioning
  - all tuples that lie in the $i$th range are sent to processor $P_i$
  - $P_i$ stores the tuples it received temporarily on disk $D_i$.
- This step requires I/O and communication overhead.
- Each processor $P_i$ sorts its partition of the relation locally.
- Each processors executes same operation (sort) in parallel with other processors, without any interaction with the others (data parallelism).
- Final merge operation is trivial: range-partitioning ensures that, for $1 \leq j \leq m$, the key values in processor $P_i$ are all less than the key values in $P_j$. 
Parallel External Sort-Merge

- Assume the relation has already been partitioned among disks $D_0$, ..., $D_{n-1}$ (in whatever manner).
- Each processor $P_i$ locally sorts the data on disk $D_i$.
- The sorted runs on each processor are then merged to get the final sorted output.
- Parallelize the merging of sorted runs as follows:
  - The sorted partitions at each processor $P_i$ are range-partitioned across the processors $P_0$, ..., $P_{m-1}$.
  - Each processor $P_i$ performs a merge on the streams as they are received, to get a single sorted run.
  - The sorted runs on processors $P_0$, ..., $P_{m-1}$ are concatenated to get the final result.
Parallel Join

- The join operation requires pairs of tuples to be tested to see if they satisfy the join condition, and if they do, the pair is added to the join output.
- Parallel join algorithms attempt to split the pairs to be tested over several processors. Each processor then computes part of the join locally.
- In a final step, the results from each processor can be collected together to produce the final result.
Partitioned Join

- For equi-joins and natural joins, it is possible to *partition* the two input relations across the processors, and compute the join locally at each processor.

- Let $r$ and $s$ be the input relations, and we want to compute $r \bowtie_{A=s.B} s$.

- $r$ and $s$ each are partitioned into $n$ partitions, denoted $r_0, r_1, \ldots, r_{n-1}$ and $s_0, s_1, \ldots, s_{n-1}$.

- Can use either *range partitioning* or *hash partitioning*.

- $r$ and $s$ must be partitioned on their join attributes $r.A$ and $s.B$, using the same range-partitioning vector or hash function.

- Partitions $r_i$ and $s_i$ are sent to processor $P_i$.

- Each processor $P_i$ locally computes $r_i \bowtie_{i.A=s_i.B} s_i$. Any of the standard join methods can be used.
Partitioned Join (Cont.)
Fragment-and-Replicate Join

- Partitioning not possible for some join conditions
  - E.g., non-equijoin conditions, such as \( r.A > s.B \).
- For joins were partitioning is not applicable, parallelization can be accomplished by **fragment and replicate** technique
  - Depicted on next slide
- Special case – **asymmetric fragment-and-replicate**:
  - One of the relations, say \( r \), is partitioned; any partitioning technique can be used.
  - The other relation, \( s \), is replicated across all the processors.
  - Processor \( P_i \) then locally computes the join of \( r_i \) with all of \( s \) using any join technique.
Depiction of Fragment-and-Replicate Joins

(a) Asymmetric fragment and replicate

(b) Fragment and replicate
Fragment-and-Replicate Join (Cont.)

- General case: reduces the sizes of the relations at each processor.
  - \( r \) is partitioned into \( n \) partitions, \( r_0, r_1, \ldots, r_{n-1} \); \( s \) is partitioned into \( m \) partitions, \( s_0, s_1, \ldots, s_{m-1} \).
  - Any partitioning technique may be used.
  - There must be at least \( m \times n \) processors.
  - Label the processors as \( P_{0,0}, P_{0,1}, \ldots, P_{0,m-1}, P_{1,0}, \ldots, P_{n-1,m-1} \).
  - \( P_{i,j} \) computes the join of \( r_i \) with \( s_j \). In order to do so, \( r_i \) is replicated to \( P_{i,0}, P_{i,1}, \ldots, P_{i,m-1} \), while \( s_i \) is replicated to \( P_{0,i}, P_{1,i}, \ldots, P_{n-1,i} \).
  - Any join technique can be used at each processor \( P_{i,j} \).
Both versions of fragment-and-replicate work with any join condition, since every tuple in $r$ can be tested with every tuple in $s$.

Usually has a higher cost than partitioning, since one of the relations (for asymmetric fragment-and-replicate) or both relations (for general fragment-and-replicate) have to be replicated.

Sometimes asymmetric fragment-and-replicate is preferable even though partitioning could be used.

- E.g., say $s$ is small and $r$ is large, and already partitioned. It may be cheaper to replicate $s$ across all processors, rather than repartition $r$ and $s$ on the join attributes.
Partitioned Parallel Hash-Join

Parallelizing partitioned hash join:

- Assume $s$ is smaller than $r$ and therefore $s$ is chosen as the build relation.

- A hash function $h_1$ takes the join attribute value of each tuple in $s$ and maps this tuple to one of the $n$ processors.

- Each processor $P_i$ reads the tuples of $s$ that are on its disk $D_i$, and sends each tuple to the appropriate processor based on hash function $h_1$. Let $s_i$ denote the tuples of relation $s$ that are sent to processor $P_i$.

- As tuples of relation $s$ are received at the destination processors, they are partitioned further using another hash function, $h_2$, which is used to compute the hash-join locally. (Cont.)
Partitioned Parallel Hash-Join (Cont.)

- Once the tuples of $s$ have been distributed, the larger relation $r$ is redistributed across the $m$ processors using the hash function $h_1$
  - Let $r_i$ denote the tuples of relation $r$ that are sent to processor $P_i$.
- As the $r$ tuples are received at the destination processors, they are repartitioned using the function $h_2$
  - (just as the probe relation is partitioned in the sequential hash-join algorithm).
- Each processor $P_i$ executes the build and probe phases of the hash-join algorithm on the local partitions $r_i$ and $s$ of $r$ and $s$ to produce a partition of the final result of the hash-join.
- Note: Hash-join optimizations can be applied to the parallel case
  - e.g., the hybrid hash-join algorithm can be used to cache some of the incoming tuples in memory and avoid the cost of writing them and reading them back in.
Parallel Nested-Loop Join

- Assume that
  - relation $s$ is much smaller than relation $r$ and that $r$ is stored by partitioning.
  - there is an index on a join attribute of relation $r$ at each of the partitions of relation $r$.

- Use asymmetric fragment-and-replicate, with relation $s$ being replicated, and using the existing partitioning of relation $r$.  

- Each processor $P_j$ where a partition of relation $s$ is stored reads the tuples of relation $s$ stored in $D_j$, and replicates the tuples to every other processor $P_i$.
  - At the end of this phase, relation $s$ is replicated at all sites that store tuples of relation $r$.

- Each processor $P_i$ performs an indexed nested-loop join of relation $s$ with the $i^{th}$ partition of relation $r$. 
Other Relational Operations

Selection $\sigma_\theta(r)$

- If $\theta$ is of the form $a_i = v$, where $a_i$ is an attribute and $v$ a value.
  - If $r$ is partitioned on $a_i$ the selection is performed at a single processor.
- If $\theta$ is of the form $l \leq a_i \leq u$ (i.e., $\theta$ is a range selection) and the relation has been range-partitioned on $a_i$
  - Selection is performed at each processor whose partition overlaps with the specified range of values.
- In all other cases: the selection is performed in parallel at all the processors.
Other Relational Operations (Cont.)

- **Duplicate elimination**
  - Perform by using either of the parallel sort techniques
    - eliminate duplicates as soon as they are found during sorting.
  - Can also partition the tuples (using either range- or hash-partitioning) and perform duplicate elimination locally at each processor.

- **Projection**
  - Projection without duplicate elimination can be performed as tuples are read in from disk in parallel.
  - If duplicate elimination is required, any of the above duplicate elimination techniques can be used.
Grouping/Aggregation

- Partition the relation on the grouping attributes and then compute the aggregate values locally at each processor.
- Can reduce cost of transferring tuples during partitioning by partly computing aggregate values before partitioning.
- Consider the **sum** aggregation operation:
  - Perform aggregation operation at each processor $P_i$ on those tuples stored on disk $D_i$
    - results in tuples with partial sums at each processor.
  - Result of the local aggregation is partitioned on the grouping attributes, and the aggregation performed again at each processor $P_i$ to get the final result.
- Fewer tuples need to be sent to other processors during partitioning.
Cost of Parallel Evaluation of Operations

- If there is no skew in the partitioning, and there is no overhead due to the parallel evaluation, expected speed-up will be $\frac{1}{n}$

- If skew and overheads are also to be taken into account, the time taken by a parallel operation can be estimated as

  $$T_{\text{part}} + T_{\text{asm}} + \max (T_0, T_1, \ldots, T_{n-1})$$

- $T_{\text{part}}$ is the time for partitioning the relations
- $T_{\text{asm}}$ is the time for assembling the results
- $T_i$ is the time taken for the operation at processor $P_i$
  - this needs to be estimated taking into account the skew, and the time wasted in contentions.
Interoperator Parallelism

- Pipelined parallelism
  - Consider a join of four relations
    - \( r_1 \bowtie r_2 \bowtie r_3 \bowtie r_4 \)
  - Set up a pipeline that computes the three joins in parallel
    - Let P1 be assigned the computation of \( \text{temp1} = r_1 \bowtie r_2 \)
    - And P2 be assigned the computation of \( \text{temp2} = \text{temp1} \bowtie r_3 \)
    - And P3 be assigned the computation of \( \text{temp2} \bowtie r_4 \)
  - Each of these operations can execute in parallel, sending result tuples it computes to the next operation even as it is computing further results
    - Provided a pipelineable join evaluation algorithm (e.g., indexed nested loops join) is used
Factors Limiting Utility of Pipeline Parallelism

- Pipeline parallelism is useful since it avoids writing intermediate results to disk.
- Useful with small number of processors, but does not scale up well with more processors. One reason is that pipeline chains do not attain sufficient length.
- Cannot pipeline operators which do not produce output until all inputs have been accessed (e.g., aggregate and sort).
- Little speedup is obtained for the frequent cases of skew in which one operator's execution cost is much higher than the others.
Independent Parallelism

- **Independent parallelism**
  - Consider a join of four relations
    \[ r_1 \bowtie r_2 \bowtie r_3 \bowtie r_4 \]
    - Let \( P_1 \) be assigned the computation of \( \text{temp1} = r_1 \bowtie r_2 \)
    - And \( P_2 \) be assigned the computation of \( \text{temp2} = r_3 \bowtie r_4 \)
    - And \( P_3 \) be assigned the computation of \( \text{temp1} \bowtie \text{temp2} \)
    - \( P_1 \) and \( P_2 \) can work **independently in parallel**
    - \( P_3 \) has to wait for input from \( P_1 \) and \( P_2 \)
      - Can pipeline output of \( P_1 \) and \( P_2 \) to \( P_3 \), combining independent parallelism and pipelined parallelism
  - Does not provide a high degree of parallelism
    - useful with a lower degree of parallelism.
    - less useful in a highly parallel system.