

CMSC 430
Introduction to Compilers
Fall 2015

Lexing and Parsing

Overview

- Compilers are roughly divided into two parts
 - Front-end — deals with surface syntax of the language
 - Back-end — analysis and code generation of the output of the front-end



- Lexing and Parsing translate source code into form more amenable for analysis and code generation
- Front-end also may include certain kinds of semantic analysis, such as symbol table construction, type checking, type inference, etc.

Lexing vs. Parsing

- Language grammars usually split into two levels
 - Tokens — the “words” that make up “parts of speech”
 - Ex: Identifier `[a-zA-Z_]+`
 - Ex: Number `[0-9]+`
 - Programs, types, statements, expressions, declarations, definitions, etc — the “phrases” of the language
 - Ex: `if (expr) expr;`
 - Ex: `def id(id, ..., id) expr end`
- Tokens are identified by the lexer
 - Regular expressions
- Everything else is done by the parser
 - Uses grammar in which tokens are primitives
 - Implementations can look inside tokens where needed

Lexing vs. Parsing (cont'd)

- Lexing and parsing often produce abstract syntax tree as a result
 - For efficiency, some compilers go further, and directly generate intermediate representations
- Why separate lexing and parsing from the rest of the compiler?
- Why separate lexing and parsing from each other?

Parsing theory

- Goal of parsing: Discovering a parse tree (or derivation) from a sentence, or deciding there is no such parse tree
- There's an alphabet soup of parsers
 - Cocke-Younger-Kasami (CYK) algorithm; Earley's Parser
 - Can parse *any* context-free grammar (but inefficient)
 - LL(k)
 - top-down, parses input left-to right (first L), produces a leftmost derivation (second L), k characters of lookahead
 - LR(k)
 - bottom-up, parses input left-to-right (L), produces a rightmost derivation (R), k characters of lookahead
- We will study only some of this theory
 - But we'll start more concretely

Parsing practice

- Yacc and lex — most common ways to write parsers
 - yacc = “yet another compiler compiler” (but it makes parsers)
 - lex = lexical analyzer (makes lexers/tokenizers)
- These are available for most languages
 - bison/flex — GNU versions for C/C++
 - ocamlyacc/ocamllex — what we’ll use in this class

Example: Arithmetic expressions

- High-level grammar:
 - $E \rightarrow E + E \mid n \mid (E)$
- What should the tokens be?
 - Typically they are the terminals in the grammar
 - $\{+, (,), n\}$
 - Notice that n itself represents a set of values
 - Lexers use *regular expressions* to define tokens
 - But what will a typical input actually look like?

1	+	2	+	\n	(	3		+		4	2	)	eof
---	---	---	---	----	---	---	--	---	--	---	---	---	-----

- We probably want to allow for whitespace
 - Notice not included in high-level grammar: lexer can discard it
- Also need to know when we reach the end of the file
 - The parser needs to know when to stop

Lexing with ocamllex (.mll)

```
(* Slightly simplified format *)
{ header }
rule entrypoint = parse
    regexp_1 { action_1 }
    | ...
    | regexp_n { action_n }
and ...
{ trailer }
```

- Compiled to .ml output file
 - **header** and **trailer** are inlined into output file as-is
 - **regexps** are combined to form one (big!) finite automaton that recognizes the union of the regular expressions
 - Finds *longest* possible match in the case of multiple matches
 - Generated regexp matching function is called **entrypoint**

Lexing with ocamllex (.mll)

```
(* Slightly simplified format *)
{ header }
rule entrypoint = parse
    regexp_1 { action_1 }
    | ...
    | regexp_n { action_n }
and ...
{ trailer }
```

- When match occurs, generated **entrypoint** function returns value in corresponding action
 - If we are lexing for **ocamlyacc**, then we'll return tokens that are defined in the **ocamlyacc** input grammar

Example

```
{
  open Ex1_parser
  exception Eof
}
rule token = parse
  [' ' '\t' '\r']      { token lexbuf }  (* skip blanks *)
| ['\n' ]              { EOL }
| ['0'-'9']+ as lxm    { INT(int_of_string lxm) }
| '+'                  { PLUS }
| '('                  { LPAREN }
| ')'                  { RPAREN }
| eof                  { raise Eof }
```

```
(* token definition from Ex1_parser *)
type token =
  | INT of (int)
  | EOL
  | PLUS
  | LPAREN
  | RPAREN
```

Generated code

```
# 1 "ex1_lexer.mll"  (* line directives for error msgs *)

open Ex1_parser
exception Eof

# 7 "ex1_lexer.ml"
let __ocaml_lex_tables = {...}  (* table-driven automaton *)
let rec token lexbuf = ...  (* the generated matching fn *)
```

- You don't need to understand the generated code
 - But you should understand it's not magic
- Uses [Lexing](#) module from OCaml standard lib
- Notice that [token](#) rule was compiled to [token](#) fn
 - Mysterious [lexbuf](#) from before is the argument to [token](#)
 - Type can be examined in [Lexing](#) module ocamlloc

Lexer limitations

- Automata limited to 32767 states
 - Can be a problem for languages with lots of keywords

```
rule token = parse
  "keyword_1"    { ... }
| "keyword_2"    { ... }
| ...
| "keyword_n" { ... }
| ['A'-'Z' 'a'-'z'] ['A'-'Z' 'a'-'z' '0'-'9' '_' ] * as id
  { IDENT id}
```

- Solution?

Parsing

- Now we can build a parser that works with lexemes (tokens) from [token.mll](#)
 - Recall from 330 that parsers work by consuming one character at a time off input while building up parse tree
 - Now the input stream will be tokens, rather than chars

1	+	2	+	\n	(	3		+		4	2	)	eof
---	---	---	---	----	---	---	--	---	--	---	---	---	-----

INT(1)	PLUS	INT(2)	PLUS	LPAREN	INT(3)	PLUS	INT(42)	RPAREN	eof
--------	------	--------	------	--------	--------	------	---------	--------	-----

- Notice parser doesn't need to worry about whitespace, deciding what's an [INT](#), etc

Suitability of Grammar

- Problem: our grammar is ambiguous
 - $E \rightarrow E + E \mid n \mid (E)$
 - Exercise: find an input that shows ambiguity
- There are parsing technologies that can work with ambiguous grammars
 - But they'll provide multiple parses for ambiguous strings, which is probably not what we want
- Solution: remove ambiguity
 - One way to do this from 330:
 - $E \rightarrow T \mid E + T$
 - $T \rightarrow n \mid (E)$

Parsing with ocaml yacc (.mly)

```
%{  
  header  
%}  
  declarations  
%%  
  rules  
%%  
  trailer
```

.mly input

```
type token =  
  | INT of (int)  
  | EOL  
  | PLUS  
  | LPAREN  
  | RPAREN  
  
val main :  
  (Lexing.lexbuf -> token) ->  
    Lexing.lexbuf -> int
```

.mli output

- Compiled to .ml and .mli files
 - .mli file defines `token` type and entry point `main` for parsing
 - Notice first arg to `main` is a fn from a `lexbuf` to a `token`, i.e., the function generated from a .mll file!

Parsing with ocaml yacc (.mly)

```
%{  
  header  
%}  
  declarations  
%%  
  rules  
%%  
  trailer
```

.mly input

```
(* header *)  
type token = ...  
...  
let yytables = ...  
(* trailer *)
```

.ml output

- .ml file uses **Parsing** library to do most of the work
 - **header** and **trailer** copied direct to output
 - **declarations** lists tokens and some other stuff
 - **rules** are the productions of the grammar
 - Compiled to **yytables**; this is a table-driven parser Also include *actions* that are executed as parser executes
 - We'll see an example next

Actions

- In practice, we don't just want to check whether an input parses; we also want to do something with the result
 - E.g., we might build an AST to be used later in the compiler
- Thus, each production in `ocamlyacc` is associated with an *action* that produces a result we want
- Each rule has the format
 - `lhs: rhs {act}`
 - When parser uses a production `lhs → rhs` in finding the parse tree, it runs the code in `act`
 - The code in `act` can refer to results computed by actions of other non-terminals in `rhs`, or token values from terminals in `rhs`

Example

```
%token <int> INT
%token EOL PLUS LPAREN RPAREN
%start main          /* the entry point */
%type <int> main
%%
main:
| expr EOL           { $1 }          (* 1 *)
expr:
| term               { $1 }          (* 2 *)
| expr PLUS term     { $1 + $3 }     (* 3 *)
term:
| INT                { $1 }          (* 4 *)
| LPAREN expr RPAREN { $2 }          (* 5 *)
```

- Several kinds of declarations:
 - %token — define a token or tokens used by lexer
 - %start — define start symbol of the grammar
 - %type — specify type of value returned by actions

Actions, in action

INT(1)	PLUS	INT(2)	PLUS	LPAREN	INT(3)	PLUS	INT(42)	RPAREN	eof
--------	------	--------	------	--------	--------	------	---------	--------	-----

. + 2 + (3 + 4 2) \$
term[1]. + 2 + (3 + 4 2) \$
expr[1]. + 2 + (3 + 4 2) \$
expr[1] + term[2]. + (3 + 4 2) \$
expr[3]. + (3 + 4 2) \$
expr[3] + (term[3]. + 4 2) \$
expr[3] + (expr[3]. + 4 2) \$
expr[3] + (expr[3] + term[42].) \$
expr[3] + (expr[45].) \$
expr[3] + term[45]. \$
expr[48]. \$
main[48]

```

main:
| expr EOL           { $1 }
expr:
| term               { $1 }
| expr PLUS term     { $1 + $3 }
term:
| INT                { $1 }
| LPAREN expr RPAREN { $2 }

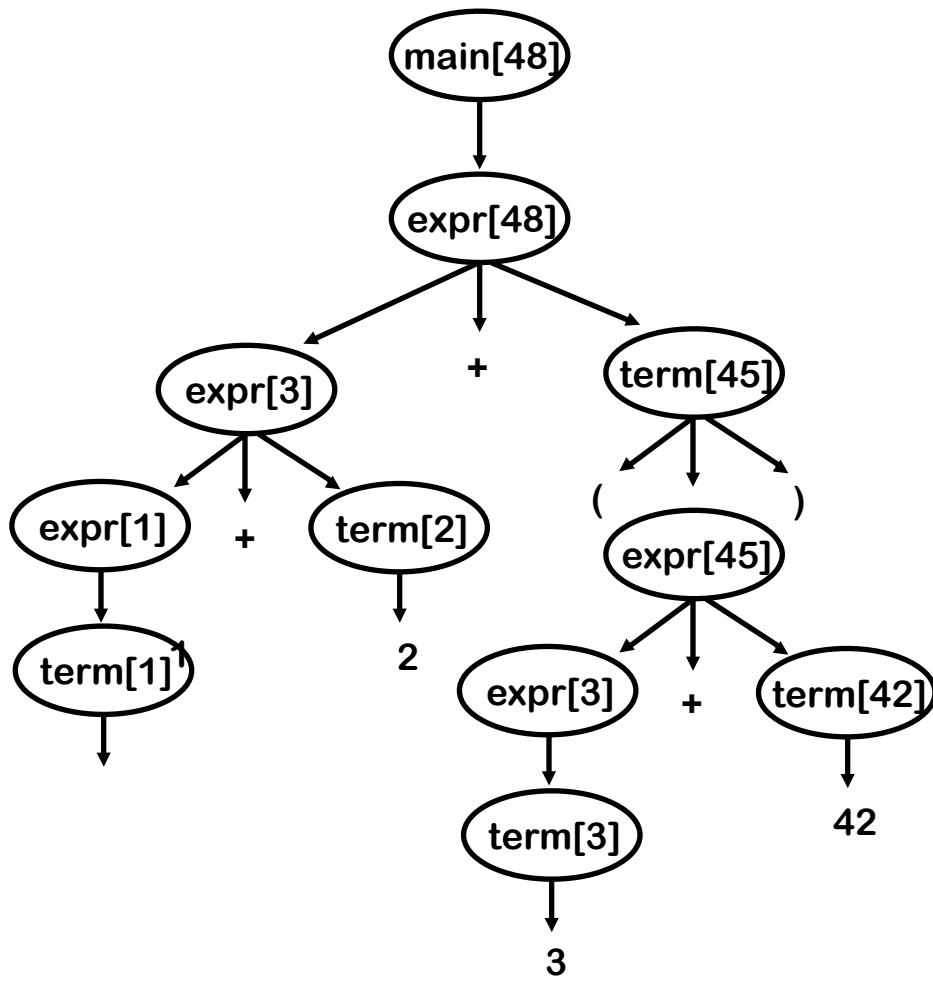
```

- The “.” indicates where we are in the parse
 - We’ve skipped several intermediate steps here, to focus only on actions
 - (Details next)

Actions, in action

INT(1)	PLUS	INT(2)	PLUS	LPAREN	INT(3)	PLUS	INT(42)	RPAREN	eof
--------	------	--------	------	--------	--------	------	---------	--------	-----

```
main:
| expr EOL           { $1 }
expr:
| term               { $1 }
| expr PLUS term     { $1 + $3 }
term:
| INT                 { $1 }
| LPAREN expr RPAREN { $2 }
```



Invoking lexer/parser

```
try
  let lexbuf = Lexing.from_channel stdin in
  while true do
    let result = Ex1_parser.main Ex1_lexer.token lexbuf in
    print_int result; print_newline(); flush stdout
  done
with Ex1_lexer.Eof ->
  exit 0
```

- Tip: can also use `Lexing.from_string` and `Lexing.from_function`

Terminology review

- Derivation
 - A sequence of steps using the productions to go from the start symbol to a string
- Rightmost (leftmost) derivation
 - A derivation in which the rightmost (leftmost) nonterminal is rewritten at each step
- Sentential form
 - A sequence of terminals and non-terminals derived from the start-symbol of the grammar with 0 or more reductions
 - I.e., some intermediate step on the way from the start symbol to a string in the language of the grammar
- Right- (left-)sentential form
 - A sentential form from a rightmost (leftmost) derivation
- FIRST(α)
 - Set of initial symbols of strings derived from α

Bottom-up parsing

- ocaml yacc builds a bottom-up parser
 - Builds derivation from input back to start symbol

$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \dots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \text{input}$
← bottom-up

- To reduce γ_i to γ_{i-1}
 - Find production $A \rightarrow \beta$ where β is in γ_i , and replace β with A
- In terms of parse tree, working from leaves to root
 - Nodes with no parent in a partial tree form its *upper fringe*
 - Since each replacement of β with A shrinks upper fringe, we call it a reduction.
- Note: need not actually build parse tree
 - $|\text{parse tree nodes}| = |\text{input}| + |\text{reductions}|$

Bottom-up parsing, illustrated

LR(1) parsing

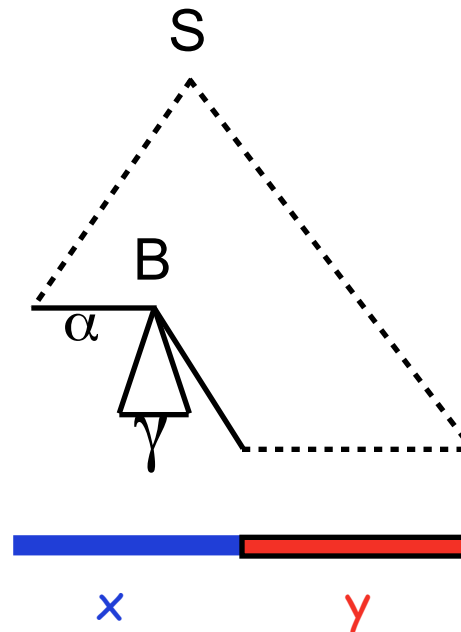
- Scan input left-to-right
- Rightmost derivation
- 1 token lookahead

$$S \Rightarrow^* \alpha B \textcolor{red}{y} \Rightarrow \alpha \gamma \textcolor{red}{y} \Rightarrow^* \textcolor{blue}{x} \textcolor{red}{y}$$

rule $B \rightarrow \gamma$



Upper fringe: solid
Yet to be parsed: dashed



Bottom-up parsing, illustrated

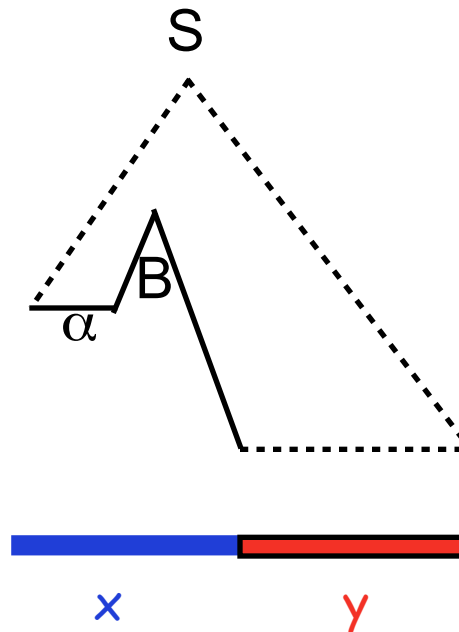
LR(1) parsing

- Scan input left-to-right
- Rightmost derivation
- 1 token lookahead

$$S \Rightarrow^* \alpha B \textcolor{red}{y} \Rightarrow \alpha \gamma \textcolor{red}{y} \Rightarrow^* \textcolor{blue}{x} \textcolor{red}{y}$$

rule $B \rightarrow \gamma$

Upper fringe: solid
Yet to be parsed: dashed



Finding reductions

- Consider the following grammar

1. $S \rightarrow a A B e$

2. $A \rightarrow A b c$

3. $\epsilon \rightarrow b$

4. $B \rightarrow d$

Input: abbcde

Sentential Form	Production	Position
abbcde	3	2
aAbcde	2	4
aAde	4	3
aABe	1	4
S	N/A	N/A

- How do we find the next reduction?
 - How do we do this efficiently?

Handles

- Goal: Find substring β of tree's frontier that matches some production $A \rightarrow \beta$
 - (And that occurs in the rightmost derivation)
 - Informally, we call this substring β a *handle*
- Formally,
 - A *handle* of a right-sentential form γ is a pair $(A \rightarrow \beta, k)$ where
 - $A \rightarrow \beta$ is a production and k is the position in γ of β 's rightmost symbol.
 - If $(A \rightarrow \beta, k)$ is a handle, then replacing β at k with A produces the right sentential form from which γ is derived in the rightmost derivation.
 - Because γ is a right-sentential form, the substring to the right of a handle contains only terminal symbols
 - $\Rightarrow$ the parser doesn't need to scan past the handle (only lookahead)

Example

- Grammar

1. $S \rightarrow E$
2. $E \rightarrow E + T$
3. $| E - T$
4. $| T$
5. $T \rightarrow T * F$
6. $| T / F$
7. $| F$
8. $F \rightarrow n$
9. $| id$
10. $| (E)$

Production	Sentential Form	Handle (prod,k)
	S	
1	E	1,1
3	E-T	3,3
5	E-T*F	5,5
9	E-T*id	9,5
7	E-F*id	7,3
8	E-n*id	8,3
4	T-n*id	4,1
7	F-n*id	7,1
9	id-n*id	9,1

Handles for rightmost derivation of $id-n*id$

Finding reductions

- Theorem: If G is unambiguous, then every right-sentential form has a unique handle
 - If we can find those handles, we can build a derivation!
- Sketch of Proof:
 - G is unambiguous $\Rightarrow$ rightmost derivation is unique
 - $\Rightarrow$ a unique production $A \rightarrow \beta$ applied to derive γ_i from γ_{i-1}
 - and a unique position k at which $A \rightarrow \beta$ is applied
 - $\Rightarrow$ a unique handle $(A \rightarrow \beta, k)$
- This all follows from the definitions

Bottom-up handle pruning

- *Handle pruning*: discovering handle and reducing it
 - Handle pruning forms the basis for bottom-up parsing
- So, to construct a rightmost derivation

$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \dots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \text{input}$

- Apply the following simple algorithm

for $i \leftarrow n$ to 1 by -1

Find handle $(A_i \rightarrow \beta_i, k_i)$ in γ_i

Replace β_i with A_i to generate γ_{i-1}

- This takes $2n$ steps

Shift-reduce parsing algorithm

- Maintain a stack of terminals and non-terminals matched so far
 - Rightmost terminal/non-terminal on top of stack
 - Since we're building rightmost derivation, will look at top elements of stack for reductions

```
push INVALID
token ← next_token( )
repeat until (top of stack = Goal and token = EOF)
  if the top of the stack is a handle  $A \rightarrow \beta$ 
    then // reduce  $\beta$  to  $A$ 
      pop  $|\beta|$  symbols off the stack
      push  $A$  onto the stack
  else if (token  $\neq$  EOF)
    then // shift
      push token
      token ← next_token( )
  else // need to shift, but out of input
    report an error
```

Potential errors

- Can't find handle
- Reach end of file

Example

1. Shift until the top of the stack is the right end of a handle
2. Find the left end of the handle & reduce

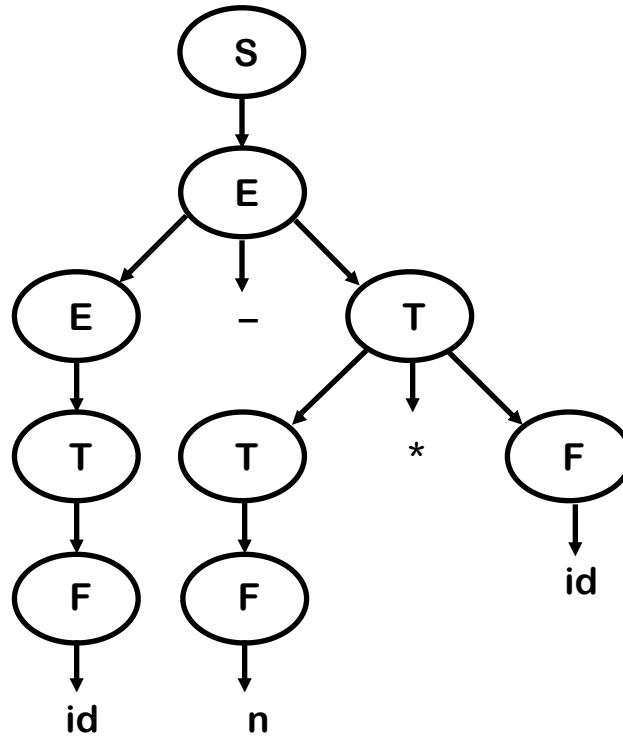
- Grammar

1. $S \rightarrow E$
2. $E \rightarrow E + T$
3. $| E - T$
4. $| T$
5. $T \rightarrow T * F$
6. $| T / F$
7. $| F$
8. $F \rightarrow n$
9. $| id$
10. $| (E)$

Shift/reduce parse of $id-n*id$

Stack	Input	Handle (prod,k)	Action
	id-n*id	none	shift
id	-n*id	9,1	reduce 9
F	-n*id	7,1	reduce 7
T	-n*id	4,1	reduce 4
E	-n*id	none	shift
E-	n*id	none	shift
E-n	*id	8,3	reduce 8
E-F	*id	7,3	reduce 7
E-T	*id	none	shift
E-T*	id	none	shift
E-T*id		9,5	reduce 9
E-T*F		5,5	reduce 5
E-T		3,3	reduce 3
E		1,1	reduce 1
S		none	accept

Parse tree for example



Algorithm actions

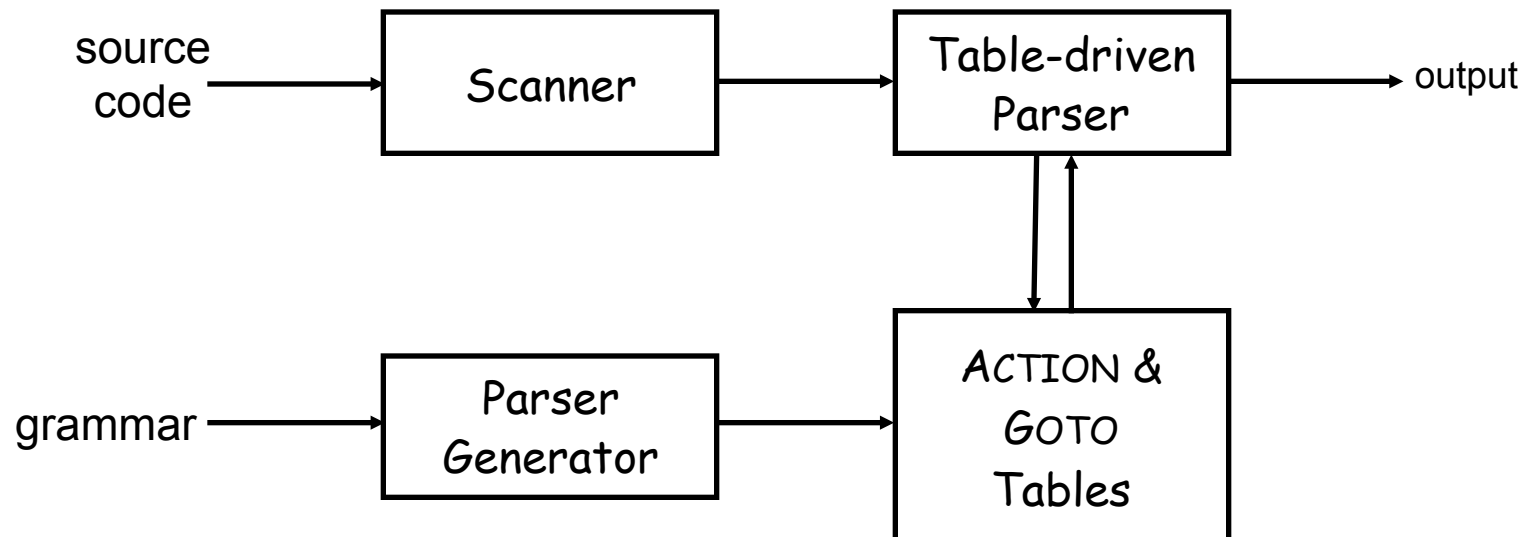
- Shift-reduce parsers have just four actions
 - **Shift** — next word is shifted onto the stack
 - **Reduce** — right end of handle is at top of stack
 - Locate left end of handle within the stack
 - Pop handle off stack and push appropriate lhs
 - **Accept** — stop parsing and report success
 - **Error** — call an error reporting/recovery routine
- Cost of operations
 - **Accept** is constant time
 - **Shift** is just a push and a call to the scanner
 - **Reduce** takes $|rhs|$ pops and 1 push
 - If handle-finding requires state, put it in the stack $\Rightarrow$ 2x work
 - **Error** depends on error recovery mechanism

Finding handles

- To be a handle, a substring of sentential form γ must :
 - Match the right hand side β of some rule $A \rightarrow \beta$
 - There must be some rightmost derivation from the start symbol that produces γ with $A \rightarrow \beta$ as the last production applied
 - $\Rightarrow$ Looking for rhs's that match strings is not good enough
- How can we know when we have found a handle?
 - LR(1) parsers use DFA that runs over stack and finds them
 - One token look-ahead determines next action (shift or reduce) in each state of the DFA.
 - A grammar is LR(1) if we can build an LR(1) parser for it
- LR(0) parsers: no look-ahead

LR(1) parsing

- Can use a set of tables to describe LR(1) parser



- ocaml yacc automates the process of building the tables
 - Standard library Parser module interprets the tables
- LR parsing invented in 1965 by Donald Knuth
- LALR parsing invented in 1969 by Frank DeRemer

LR(1) parsing algorithm

```
stack.push(INVALID); stack.push(s0);
not_found = true;
token = scanner.next_token();
do while (not_found) {
    s = stack.top();
    if ( ACTION[s,token] == "reduce A→β" ) {
        stack.popnum(2*|β|); // pop 2*|β| symbols
        s = stack.top();
        stack.push(A);
        stack.push(GOTO[s,A]);
    }
    else if ( ACTION[s,token] == "shift si" ) {
        stack.push(token); stack.push(si);
        token ← scanner.next_token();
    }
    else if ( ACTION[s,token] == "accept" && token == EOF )
        not_found = false;
    else report a syntax error and recover;
}
report success;
```

- Two tables
 - ACTION: reduce/shift/accept
 - GOTO: state to be in after reduce
- Cost
 - |input| shifts
 - |derivation| reductions
 - One accept
- Detects errors by failure to shift, reduce, or accept

Example parser table

- `ocamlyacc -v ex1_parser.mly` — produce `.output` file with parser table

state	action						goto			productions
	.	EOL	+	N	(	)	main	expr	term	
0										(special)
1				s3	s4		acc	6	7	entry → . main
2										(special)
3	r4									term → INT .
4				s3	s4			8	7	term → (. expr)
5										(special)
6		s9	s10							main → expr . EOL expr → expr . + term
7	r2									expr → term .
8			s10			s11				expr → expr . + term term → (expr .)
9	r1									main → expr EOL .
10				s3	s4				12	expr → expr + . term
11	r5									term → (expr) .
12	r3									expr → expr + term .

NB: Numbers in shift refer to state numbers

Numbers in reduction refer to production numbers

Example parse (N+N+N)

Stack	Input	Action
	N+N+N	s3
,N,3	+N+N	r4
,term,7	+N+N	r2
,expr,6	+N+N	s10
,expr,6,+,10	N+N	s3
,expr,6,+,10,N,3	+N	r4
,expr,6,+,10,term,12	+N	r3
,expr,6	+N	s10
,expr,6,+,10	N	s3
,expr,6,+,10,N,3		r4
,expr,6,+,10,term,12		r3
,expr,6		s9
,expr,6,EOL,9		r1
accept		

Example parser table (cont'd)

- Notes
 - Notice derivation is built up (bottom to top)
 - Table only contains kernel of each state
 - Apply closure operation to see all the productions in the state
- LR(1) parsing requires start symbol not on any rhs
 - Thus, ocaml yacc actually adds another production
 - `%entry% → \001 main`
 - (so the `acc` in the previous table is a slight fib)
- Values returned from actions stored on the stack
 - Reduce triggers computation of action result

Why does this work?

- Stack = upper fringe
 - So all possible handles on top of stack
 - Shift inputs until top elements of stack form a handle
- Build a handle-recognizing DFA
 - Language of handles is regular
 - ACTION and GOTO tables encode the DFA
 - Shift = DFA transition
 - Reduce = DFA accept
 - New state = GOTO[state at top of stack (after pop), lhs]
- If we can build these tables, grammar is LR(1)

LR(k) items

- An $LR(k)$ item is a pair $[P, \delta]$, where
 - P is a production $A \rightarrow \beta$ with a $\bullet$ at some position in the rhs
 - δ is a lookahead string of length $\leq k$ (words or \$)
 - The $\bullet$ in an item indicates the position of the top of the stack
- LR(1):
 - $[A \rightarrow \bullet \beta \gamma, a]$ — input so far consistent with using $A \rightarrow \beta \gamma$ immediately after symbol on top of stack
 - $[A \rightarrow \beta \bullet \gamma, a]$ — input so far consistent with using $A \rightarrow \beta \gamma$ at this point in the parse, and parser has already recognized β
 - $[A \rightarrow \beta \gamma \bullet, a]$ — parser has seen $\beta \gamma$, and lookahead of a consistent with reducing to A
- LR(1) items represent valid configurations of an LR(1) parser; DFA states are sets of LR(1) items

LR(k) items, cont'd

- Ex: $A \rightarrow BCD$ with lookahead a can yield 4 items
 - $[A \rightarrow \bullet BCD, a]$, $[A \rightarrow B \bullet CD, a]$, $[A \rightarrow BC \bullet D, a]$, $[A \rightarrow BCD \bullet, a]$
 - Notice: set of LR(1) items for a grammar is finite
- Carry lookaheads along to choose correct reduction
 - Lookahead has no direct use in $[A \rightarrow \beta \bullet \gamma, a]$
 - In $[A \rightarrow \beta \bullet, a]$, a lookahead of $a \Rightarrow$ reduction by $A \rightarrow \beta$
 - For $\{ [A \rightarrow \beta \bullet, a], [B \rightarrow \gamma \bullet \delta, b] \}$
 - Lookahead of $a \Rightarrow$ reduce to A
 - $\text{FIRST}(\delta) \Rightarrow$ shift
 - (else error)

LR(1) table construction

- States of LR(1) parser contain sets of LR(1) items
 - Initial state s_0
 - Assume S' is the start symbol of grammar, does not appear in rhs
 - (Extend grammar if necessary to ensure this)
 - $s_0 = \text{closure}([S' \rightarrow \bullet S, \$])$ ($\$ = \text{EOF}$)
 - For each s_k and each terminal/non-terminal X , compute new state $\text{goto}(s_k, X)$
 - Use $\text{closure}()$ to “fill out” kernel of new state
 - If the new state is not already in the collection, add it
 - Record all the transitions created by $\text{goto}()$
 - These become ACTION and GOTO tables
 - i.e., the handle-finding DFA
 - This process eventually reaches a fixpoint

Closure()

- $[A \rightarrow \beta \bullet B \delta, a]$ implies $[B \rightarrow \bullet \gamma, x]$ for each production with B on lhs and each $x \in \text{FIRST}(\delta a)$
 - (If you're about to see a B , you may also see a γ)

```
Closure( s )
while ( s is still changing )
   $\forall$  items  $[A \rightarrow \beta \bullet B \delta, \underline{a}] \in s$            // item with  $\bullet$  to left of nonterminal B
   $\forall$  productions  $B \rightarrow \gamma \in P$                  // all productions for B
   $\forall \underline{b} \in \text{FIRST}(\delta \underline{a})$                  // tokens appearing after B
    if  $[B \rightarrow \bullet \gamma, \underline{b}] \notin s$        // form LR(1) item w/ new lookahead
      then add  $[B \rightarrow \bullet \gamma, \underline{b}]$  to s    // add item to s if new
```

- Classic fixed-point method
- Halts because $s \subset \text{ITEMS}$ (worklist version is faster)
 - Closure “fills out” a state

Example — closure with LR(0)

$S \rightarrow E$

$E \rightarrow T + E$

$\mid T$

$T \rightarrow \text{id}$

[kernel item]
[derived item]

[$S \rightarrow \bullet E$]
[$E \rightarrow \bullet T + E$]
[$E \rightarrow \bullet T$]
[$T \rightarrow \bullet \text{id}$]

[$E \rightarrow T + \bullet E$]
[$E \rightarrow \bullet T + E$]
[$E \rightarrow \bullet T$]
[$T \rightarrow \bullet \text{id}$]

Example — closure with LR(1)

$S \rightarrow E$

$E \rightarrow T + E$

$\mid T$

$T \rightarrow \text{id}$

[kernel item]
[derived item]

[$S \rightarrow \bullet E, \$$]
[$E \rightarrow \bullet T + E, \$$]
[$E \rightarrow \bullet T, \$$]
[$T \rightarrow \bullet \text{id}, +$]
[$T \rightarrow \bullet \text{id}, \$$]

[$E \rightarrow T + \bullet E, \$$]
[$E \rightarrow \bullet T + E, \$$]
[$E \rightarrow \bullet T, \$$]
[$T \rightarrow \bullet \text{id}, +$]
[$T \rightarrow \bullet \text{id}, \$$]

Goto

- $\text{Goto}(s, x)$ computes the state that the parser would reach if it recognized an x while in state s
 - $\text{Goto}(\{ [A \rightarrow \beta \bullet X \delta, a] \}, X)$ produces $[A \rightarrow \beta X \bullet \delta, a]$
 - Should also includes $\text{closure}([A \rightarrow \beta X \bullet \delta, a])$

```
Goto( s, X )  
  new  $\leftarrow \emptyset$   
   $\forall$  items  $[A \rightarrow \beta \bullet X \delta, \underline{a}] \in s$            // for each item with  $\bullet$  to left of X  
    new  $\leftarrow$  new  $\cup [A \rightarrow \beta X \bullet \delta, \underline{a}]$  // add item with  $\bullet$  to right of X  
  return closure(new)                               // remember to compute closure!
```

- Not a fixed-point method!
- Straightforward computation
- Uses $\text{closure}()$
 - $\text{Goto}()$ moves forward

Example — goto with LR(0)

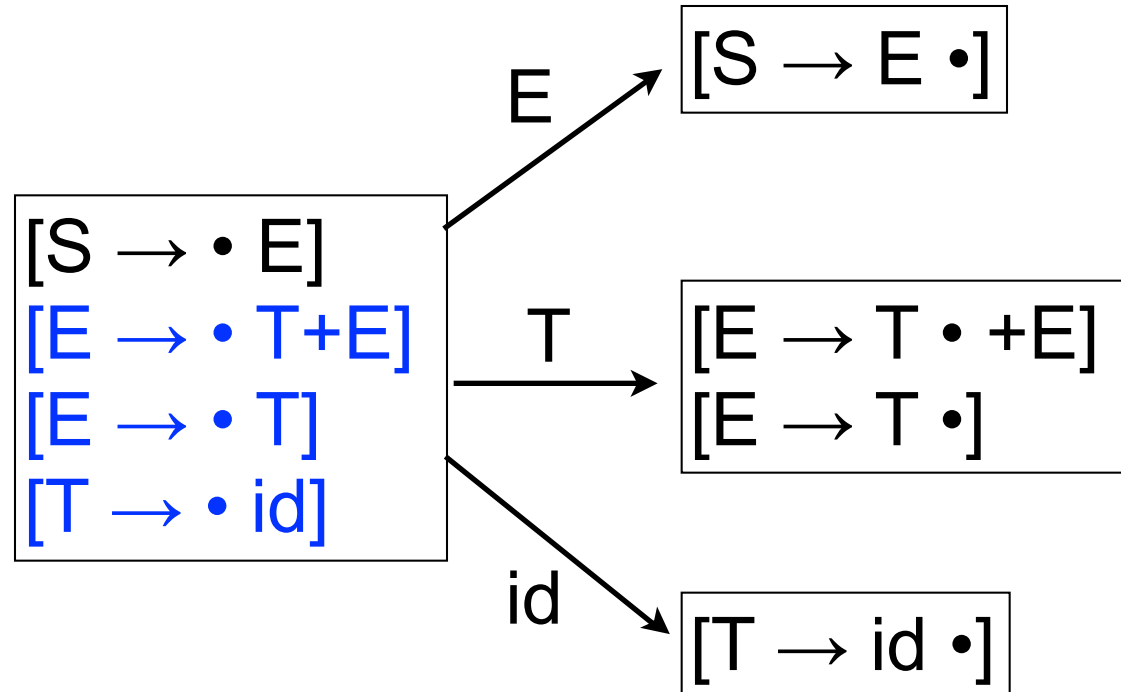
$S \rightarrow E$

$E \rightarrow T + E$

$\mid T$

$T \rightarrow \text{id}$

[kernel item]
[derived item]



Example — goto with LR(1)

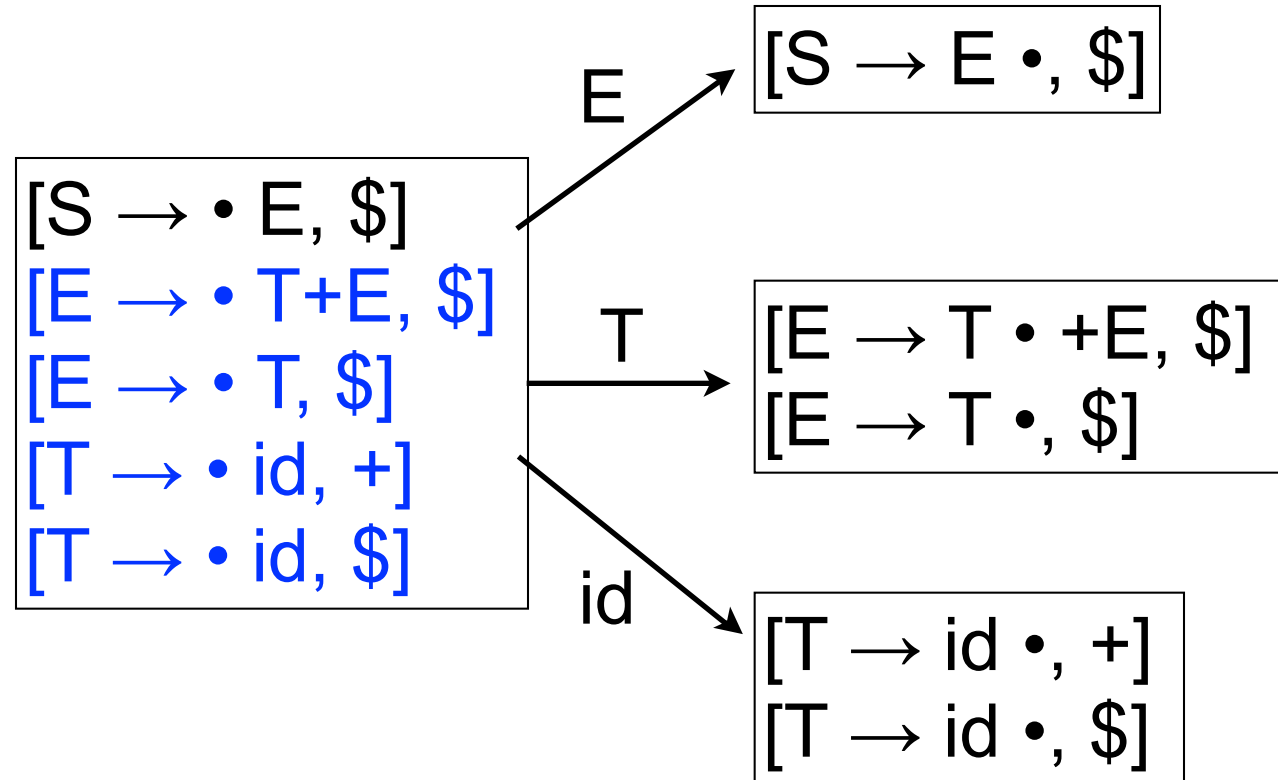
$S \rightarrow E$

$E \rightarrow T + E$

$\mid T$

$T \rightarrow \text{id}$

[kernel item]
[derived item]



Building parser states

```
cc0 ← closure ( [S' → •S, $] )  
CC ← { cc0 }  
while ( new sets are still being added to CC )  
  for each unmarked set ccj ∈ CC  
    mark ccj as processed  
    for each x following a • in an item in ccj  
      temp ← goto(ccj, x)  
      if temp ∉ CC  
        then CC ← CC ∪ { temp }  
      record transitions from ccj to temp on x
```

- CC = canonical collection (of LR(k) items)
- Fixpoint computation (worklist version)
- Loop adds to CC
 - $CC \subseteq 2^{\text{ITEMS}}$, so CC is finite

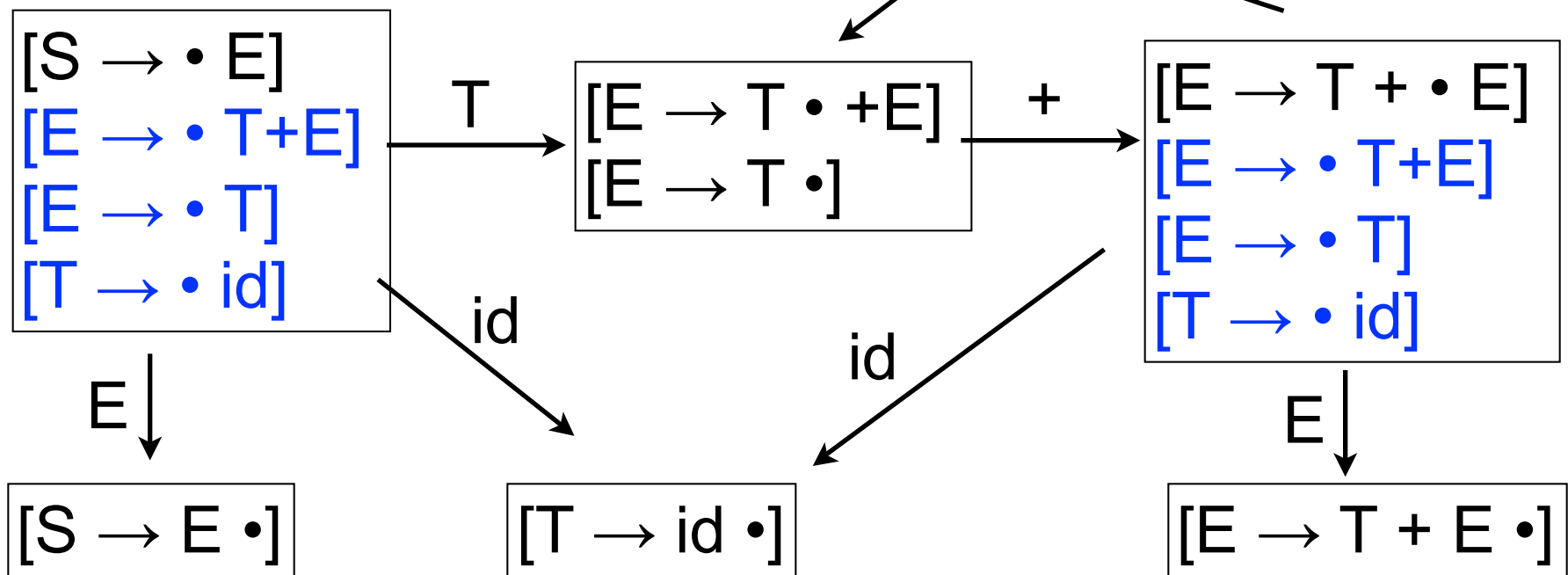
Example LR(0) states

$S \rightarrow E$

$E \rightarrow T + E$

$\mid T$

$T \rightarrow id$



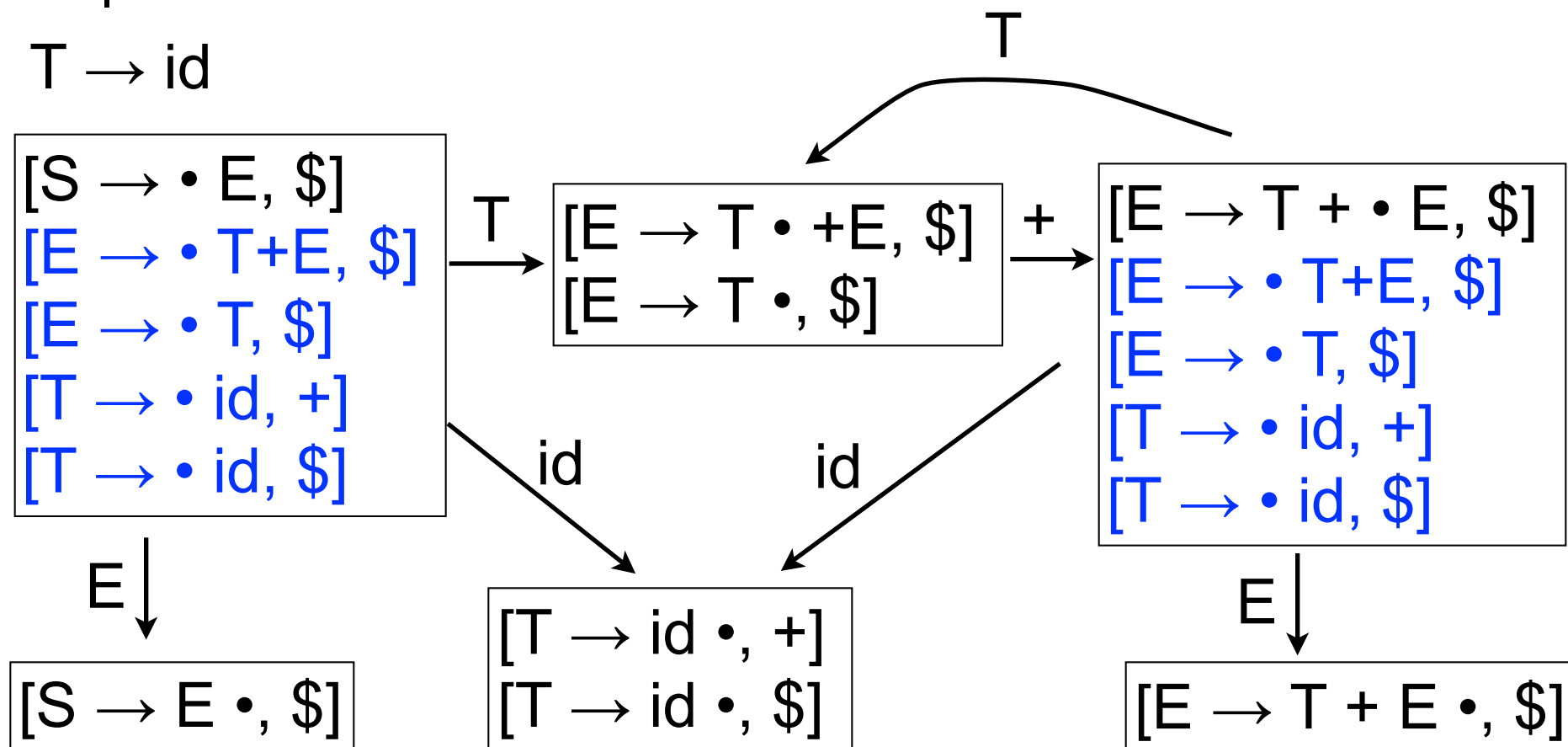
Example LR(1) states

$S \rightarrow E$

$E \rightarrow T + E$

$\mid T$

$T \rightarrow id$



Building ACTION and GOTO tables

```

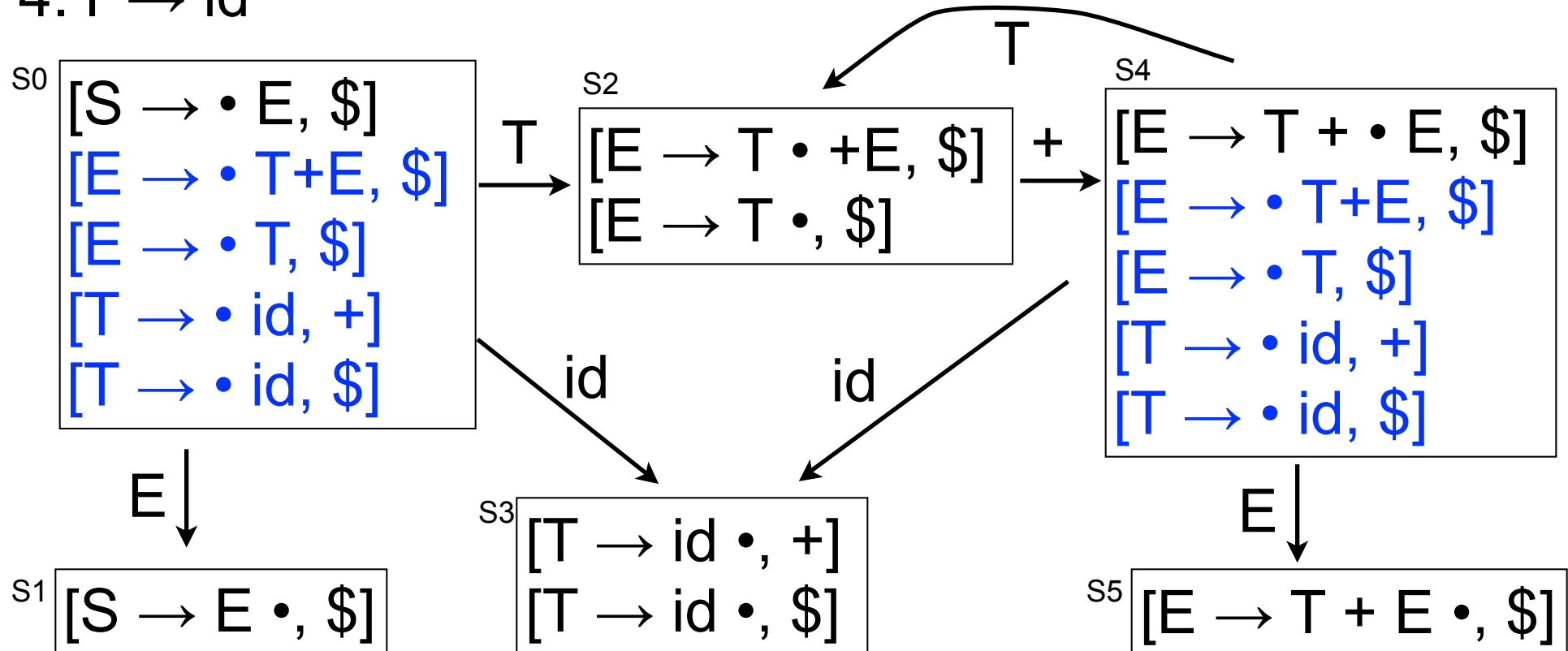
 $\forall$  set  $s_x \in S$ 
   $\forall$  item  $i \in s_x$ 
    if  $i$  is  $[A \rightarrow \beta \cdot \underline{a} \gamma, \underline{b}]$  and  $\text{goto}(s_x, \underline{a}) = s_k$ ,  $\underline{a} \in \text{terminals}$  //  $\bullet$  to left of terminal  $a$ 
      then  $\text{ACTION}[x, \underline{a}] \leftarrow \text{"shift } k"$  //  $\Rightarrow$  shift if lookahead =  $a$ 
    else if  $i$  is  $[S' \rightarrow S \cdot, \$]$  // start production done,
      then  $\text{ACTION}[x, \$] \leftarrow \text{"accept"}$  //  $\Rightarrow$  accept if lookahead =  $\$$ 
    else if  $i$  is  $[A \rightarrow \beta \cdot, \underline{a}]$  //  $\bullet$  all the way to right
      then  $\text{ACTION}[x, \underline{a}] \leftarrow \text{"reduce } A \rightarrow \beta"$  //  $\rightarrow$  production done
   $\forall n \in \text{nonterminals}$  // reduce if lookahead =  $a$ 
    if  $\text{goto}(s_x, n) = s_k$ 
      then  $\text{GOTO}[x, n] \leftarrow k$  // store transitions for nonterminals
```

- Many items generate no table entry
 - e.g., $[A \rightarrow \beta \cdot B \alpha, a]$ does not, but closure ensures that all the rhs's for B are in s_x

Ex ACTION and GOTO tables

1. $S \rightarrow E$
2. $E \rightarrow T + E$
3. $\mid T$
4. $T \rightarrow id$

	ACTION			GOTO	
	id	+	\$	E	T
S0	s3			1	2
S1			acc		
S2		s4	r3		
S3		r4	r4		
S4	s3			5	2
S5			r2		

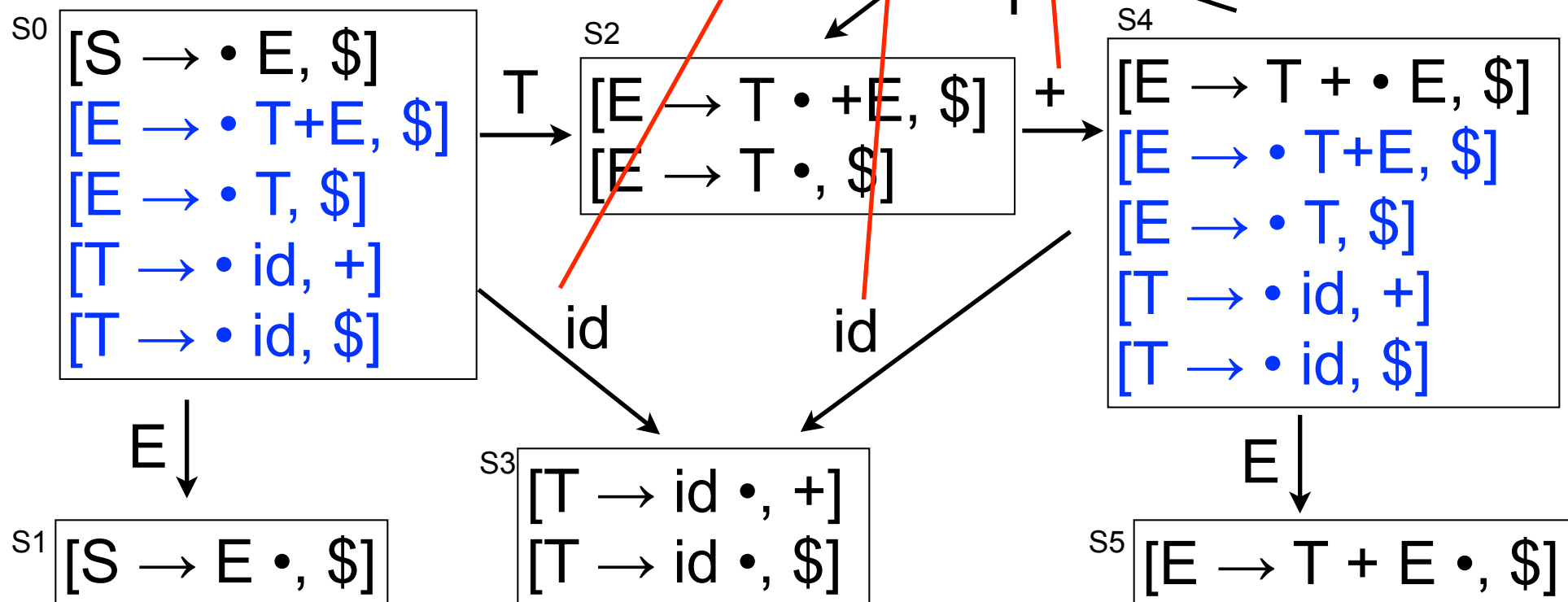


Ex ACTION and GOTO tables

1. $S \rightarrow E$
2. $E \rightarrow T + E$
3. $\mid T$
4. $T \rightarrow id$

Entries
for
shift

	ACTION			GOTO	
	id	+	\$	E	T
S0	s3			1	2
S1			acc		
S2		s4	r3		
S3		r4	r4		
S4	s3			5	2
S5			r2		

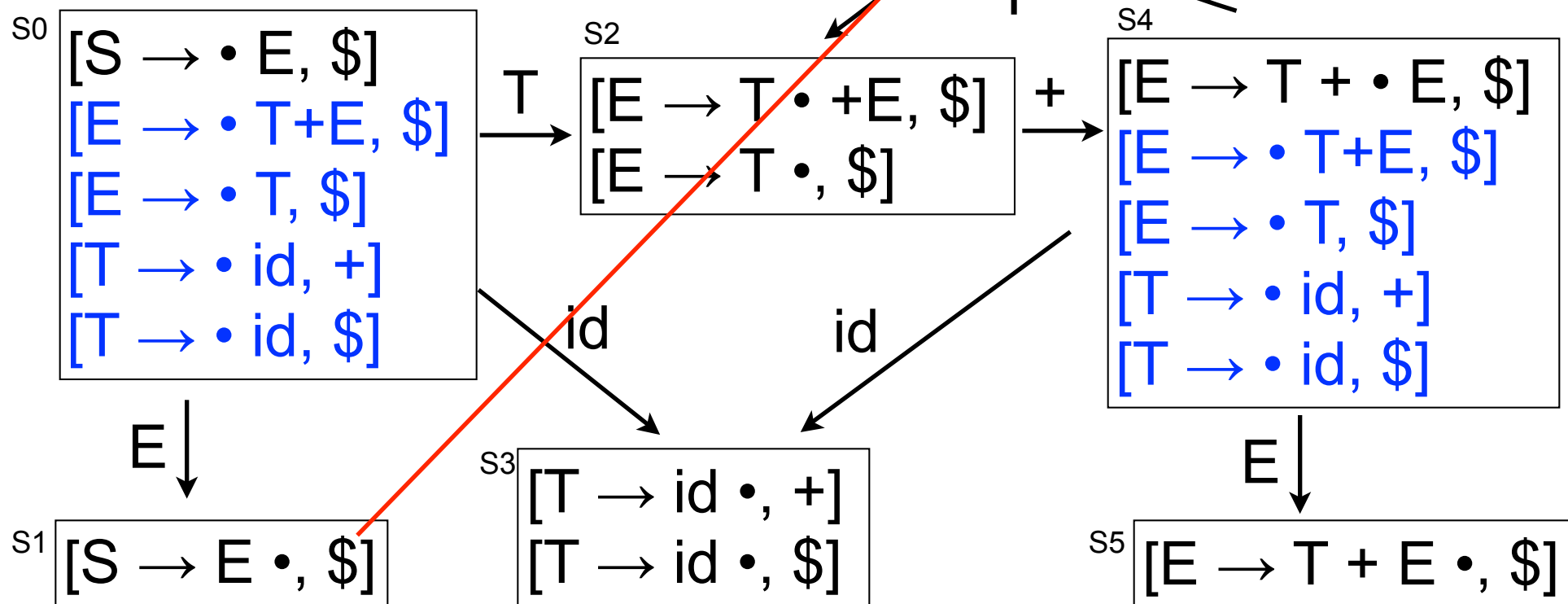


Ex ACTION and GOTO tables

1. $S \rightarrow E$
2. $E \rightarrow T + E$
3. $\mid T$
4. $T \rightarrow id$

Entry
for
accept

	ACTION			GOTO	
	id	+	\$	E	T
S0	s3			1	2
S1			acc		
S2		s4	r3		
S3		r4	r4		
S4	s3			5	2
S5			r2		

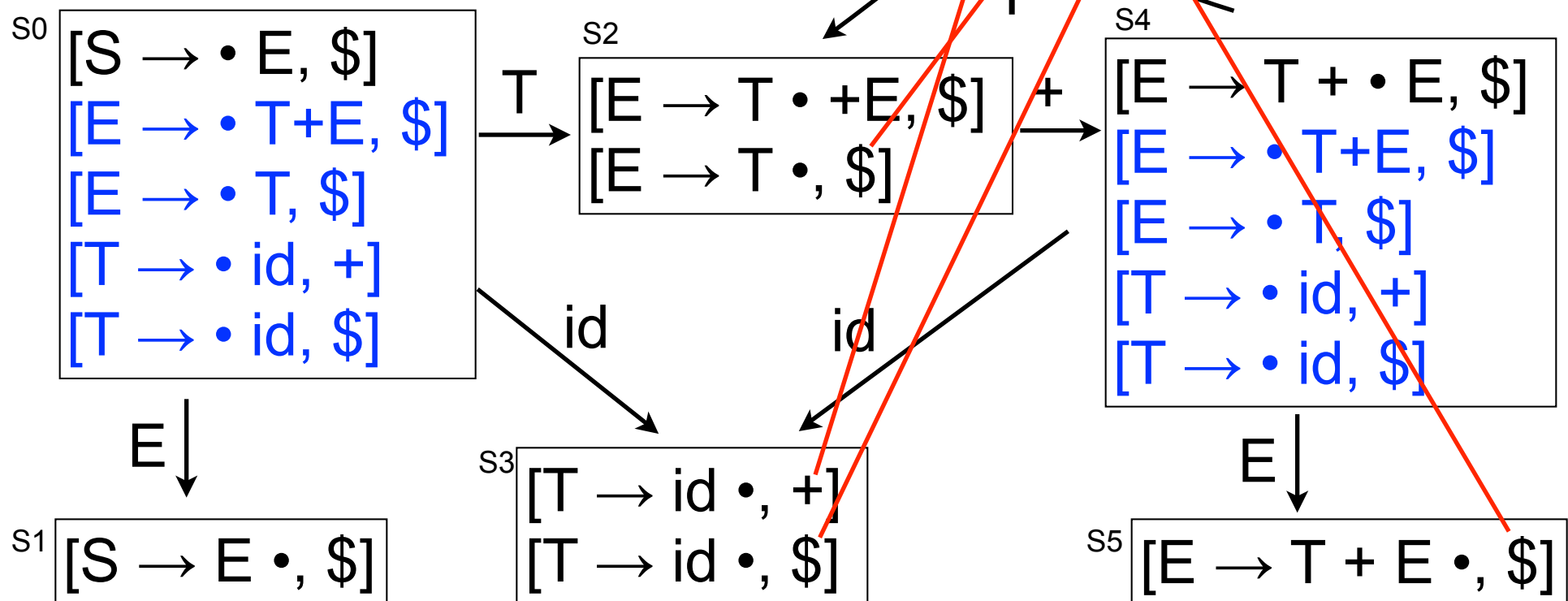


Ex ACTION and GOTO tables

1. $S \rightarrow E$
2. $E \rightarrow T + E$
3. $\mid T$
4. $T \rightarrow id$

Entries
for
reduce

	ACTION			GOTO	
	id	+	\$	E	T
S0	s3			1	2
S1			acc		
S2		s4	r3		
S3		r4	r4		
S4	s3			5	2
S5			r2		

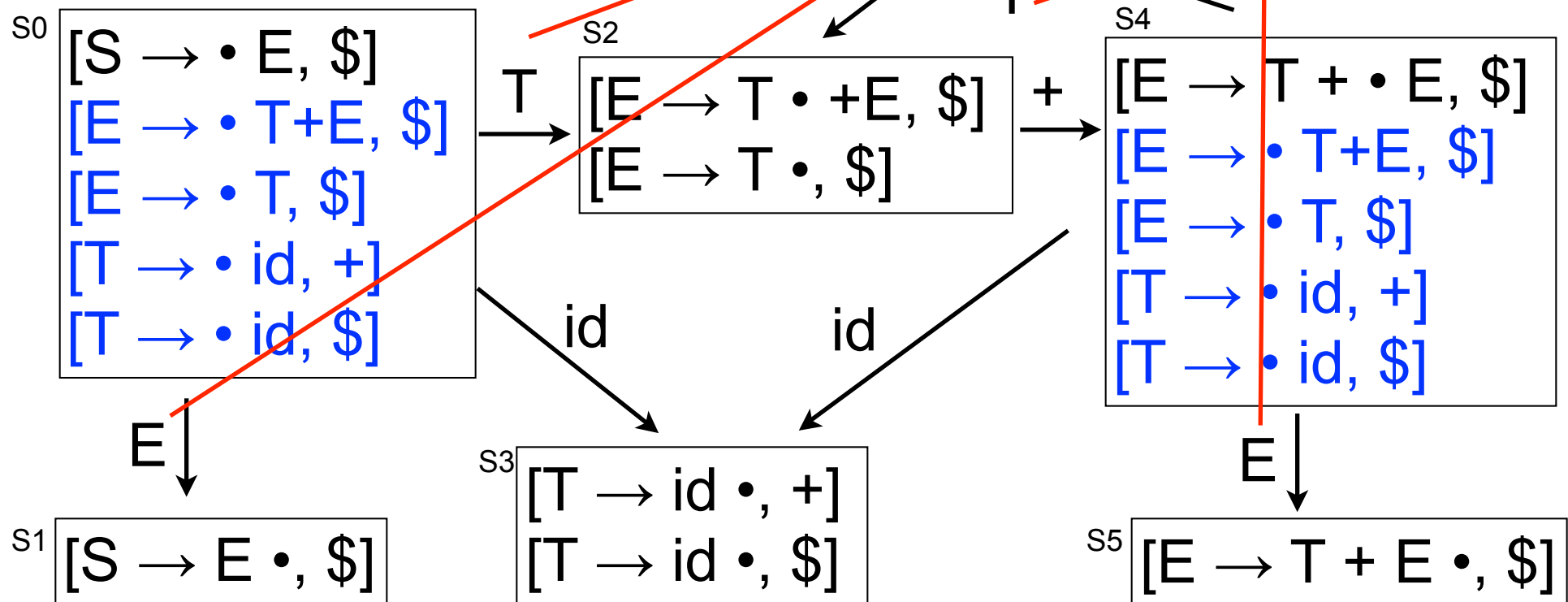


Ex ACTION and GOTO tables

1. $S \rightarrow E$
2. $E \rightarrow T + E$
3. $\mid T$
4. $T \rightarrow id$

Entries
for
GOTO

	ACTION			GOTO	
	id	+	\$	E	T
S0	s3			1	2
S1			acc		
S2		s4	r3		
S3		r4	r4		
S4	s3			5	2
S5			r2		



What can go wrong?

- What if set s contains $[A \rightarrow \beta \cdot a \gamma, b]$ and $[B \rightarrow \beta \cdot, a]$?
 - First item generates “shift”, second generates “reduce”
 - Both define $\text{ACTION}[s, a]$ — cannot do both actions
 - This is a *shift/reduce conflict*
- What if set s contains $[A \rightarrow \gamma \cdot, a]$ and $[B \rightarrow \gamma \cdot, a]$?
 - Each generates “reduce”, but with a different production
 - Both define $\text{ACTION}[s, a]$ — cannot do both reductions
 - This is called a reduce/reduce conflict
- In either case, the grammar is not LR(1)

Shift/reduce conflict

```
%token <int> INT
%token EOL PLUS LPAREN RPAREN
%start main          /* the entry point */
%type <int> main
%%
main:
| expr EOL           { $1 }
expr:
| INT                { $1 }
| expr PLUS expr     { $1 + $3 }
| LPAREN expr RPAREN { $2 }
```

- Associativity unspecified
 - Ambiguous grammars always have conflicts
 - But, some non-ambiguous grammars also have conflicts

Solving conflicts

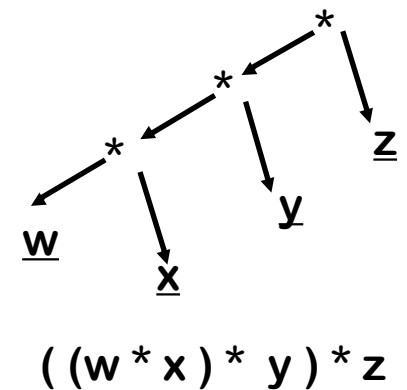
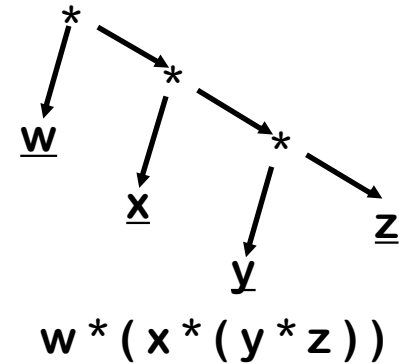
- Refactor grammar
- Specify operator precedence and associativity

```
%left PLUS MINUS      /* lowest precedence */  
%left TIMES DIV        /* medium precedence */  
%nonassoc UMINUS       /* highest precedence */
```

- Lots of details here
 - See “12.4.2 Declarations” at
 - <http://caml.inria.fr/pub/docs/manual-ocaml/manual026.html#htoc151>
- When comparing operator on stack with lookahead
 - Shift if lookahead has higher prec OR same prec, right assoc
 - Reduce if lookahead has lower prec OR same prec, left assoc
- Can use smaller, simpler (ambiguous) grammars
 - Like the one we just saw

Left vs. right recursion

- Right recursion
 - Required for termination in top-down parsers
 - Produces right-associative operators
- Left recursion
 - Works fine in bottom-up parsers
 - Limits required stack space
 - Produces left-associative operators
- Rule of thumb
 - Left recursion for bottom-up parsers
 - Right recursion for top-down parsers



Reduce/reduce conflict (1)

```
%token <int> INT
%token EOL PLUS LPAREN RPAREN
%start main          /* the entry point */
%type <int> main
%%
main:
| expr EOL           { $1 }
expr:
| INT                { $1 }
| term               { $1 }
| term PLUS expr     { $1 + $3 }
term :
| INT                { $1 }
| LPAREN expr RPAREN { $2 }
```

- Often these conflicts suggest a serious problem
 - Here, there's a deep ambiguity

Reduce/reduce conflict (2)

```
%token <int> INT
%token EOL PLUS LPAREN RPAREN
%start main          /* the entry point */
%type <int> main
%%
main:
| expr EOL           { $1 }
expr:
| term1              { $1 }
| term1 PLUS PLUS expr { $1 + $4 }
| term2 PLUS expr    { $1 + $3 }
term1 :
| INT                { $1 }
| LPAREN expr RPAREN { $2 }
term2 :
| INT                { $1 }
```

- Grammar not ambiguous, but not enough lookahead to distinguish last two `expr` productions

Shrinking the tables

- Combine terminals
 - E.g., number and identifier, or + and -, or * and /
 - Directly removes a column, may remove a row
- Combine rows or columns (*table compression*)
 - Implement identical rows once and remap states
 - Requires extra indirection on each lookup
 - Use separate mapping for ACTION and for GOTO
- Use another construction algorithm
 - LALR(1) used by ocamllyacc

LALR(1) parser

- Define the *core* of a set of LR(1) items as
 - Set of LR(0) items derived by ignoring lookahead symbols

$[E \rightarrow a \bullet, b]$
$[A \rightarrow a \bullet, c]$

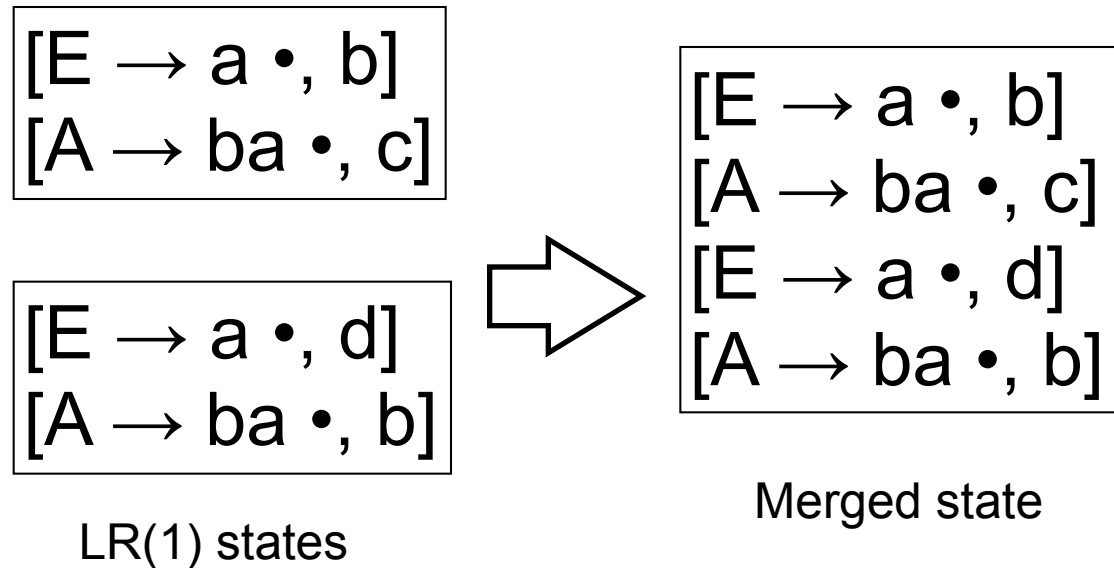
LR(1) state

$[E \rightarrow a \bullet]$
$[A \rightarrow a \bullet]$

Core

- LALR(1) parser merges two states if they have the same core
- Result
 - Potentially much smaller set of states
 - May introduce reduce/reduce conflicts
 - Will not introduce shift/reduce conflicts

LALR(1) example



- Introduces reduce/reduce conflict
 - Can reduce either $E \rightarrow a$ or $A \rightarrow ba$ for lookahead = b

LALR(1) vs. LR(1)

- Example grammar

$$\begin{aligned} S' &\rightarrow S \\ S &\rightarrow aAd \mid bBd \mid aBe \mid bAe \\ A &\rightarrow c \\ B &\rightarrow c \end{aligned}$$

- LR(0) ?
- LR(1) ?
- LALR(1) ?

LR(k) Parsers

- Properties
 - Strictly more powerful than LL(k) parsers
 - Most general non-backtracking shift-reduce parser
 - Detects error as soon as possible in left-to-right scan of input
 - Contents of stack are **viable prefixes**
 - Possible for remaining input to lead to successful parse

Error handling (lexing)

- What happens when input not handled by any lexing rule?
 - An exception gets raised
 - Better to provide more information, e.g.,

```
rule token = parse
...

| _ as lxm { Printf.printf "Illegal character %c" lxm;
              failwith "Bad input" }
```

- Even better, keep track of line numbers
 - Store in a global-ish variable (oh no!)
 - Increment as a side effect whenever `\n` recognized

Error handling (parsing)

- What happens when parsing a string not in the grammar?
 - Reject the input
 - Do we keep going, parsing more characters?
 - May cause a cascade of error messages
 - Could be more useful to programmer, if they don't need to stop at the first error message (what do you do, in practice?)
- Ocamlyacc includes a basic error recovery mechanism
 - Special token **error** may appear in rhs of production
 - Matches erroneous input, allowing recovery

Error example (1)

```
...  
expr:  
| term                { $1 }  
| expr PLUS term      { $1 + $3 }  
| error                { Printf.printf "invalid expression"; 0 }  
term: ...
```

- If unexpected input appears while trying to match `expr`, match token to `error`
 - Effectively treats token as if it is produced from `expr`
 - Triggers error action

Error example (2)

```
...  
term:  
| INT                { $1 }  
| LPAREN expr RPAREN { $2 }  
| LPAREN error RPAREN {Printf.printf "Syntax error!\n"; 0}
```

- If unexpected input appears while trying to match **term**, match tokens to **error**
 - Pop every state off the stack until **LPAREN** on top
 - Scan tokens up to **RPAREN**, and discard those, also
 - Then match **error** production

Error recovery in practice

- A very hard thing to get right!
 - Necessarily involves guessing at what malformed inputs you may see
- How useful is recovery?
 - Compilers are very fast today, so not so bad to stop at first error message, fix it, and go on
 - On the other hand, that does involve some delay
- Perhaps the most important feature is *good error messages*
 - Error recovery features useful for this, as well
 - Some compilers are better at this than others

OCamlyacc tip

- Setting OCAMLRUNPARAM=p will cause the parsing steps to be printed out as the parser runs
- (And setting OCAMLRUNPARAM=b will tell OCaml to print a stack backtrace for any thrown exceptions.)

Real programming languages

- Essentially all real programming languages don't quite work with parser generators
 - Even Java is not quite LALR(1)
- Thus, real implementations play tricks with parsing actions to resolve conflicts
- In-class exercise: C typedefs and identifier declarations/definitions

Additional Parsing Technologies

- For a long time, parsing was a “dead” field
 - Considered solved a long time ago
- Recently, people have come back to it
 - LALR parsing can have unnecessary parsing conflicts
 - LALR parsing tradeoffs more important when computers were slower and memory was smaller
- Many recent new (or new-old) parsing techniques
 - GLR — generalized LR parsing, for ambiguous grammars
 - LL(*) — ANTLR
 - Packrat parsing — for *parsing expression grammars*
 - etc...
- The input syntax to many of these looks like yacc/
lex

Designing language syntax

- Idea 1: Make it look like other, popular languages
 - Java did this (OO with C syntax)
- Idea 2: Make it look like the domain
 - There may be well-established notation in the domain (e.g., mathematics)
 - Domain experts already know that notation
- Idea 3: Measure design choices
 - E.g., ask users to perform programming (or related) task with various choices of syntax, evaluate performance, survey them on understanding
 - This is very hard to do!
- Idea 4: Make your users adapt
 - People are really good at learning...