

Practice Problems – Operational Semantics

Sample Solution

1.

$$\frac{\langle c_0, \sigma \rangle \longrightarrow \sigma' \quad \langle b, \sigma' \rangle \longrightarrow \text{false}}{\langle \text{for}(c_0; b; c_1) \ c_2, \sigma \rangle \longrightarrow \sigma'} \quad \frac{\langle c_2, \sigma_0 \rangle \longrightarrow \sigma_2 \quad \frac{\langle c_0, \sigma \rangle \longrightarrow \sigma_0 \quad \langle b, \sigma_0 \rangle \longrightarrow \text{true}}{\langle c_1, \sigma_2 \rangle \longrightarrow \sigma_3} \quad \langle \text{for}(\text{skip}; b; c_1) \ c_2, \sigma_3 \rangle \longrightarrow \sigma'}{\langle \text{for}(c_0; b; c_1) \ c_2, \sigma \rangle \longrightarrow \sigma'}$$

2. (a) $\frac{}{(\lambda x.42) \ 13 \rightarrow 42}$

(b) $\frac{\overline{(\lambda x.x) \ (\lambda y.y) \rightarrow \lambda y.y}}{((\lambda x.x) \ (\lambda y.y)) \ (\lambda z.z) \rightarrow (\lambda y.y) \ (\lambda z.z)}$

(c) $\frac{\overline{(\lambda x.\lambda y.x) \ 42 \rightarrow \lambda y.42}}{(\lambda x.x) \ ((\lambda x.\lambda y.x) \ 42) \rightarrow (\lambda x.x) \ (\lambda y.42)}$

3.

$$\frac{\text{BETA}}{(\lambda x.e) \rightarrow (\lambda x.e)} \quad \frac{\text{INT}}{n \rightarrow n} \quad \frac{\text{APP} \quad \frac{e_1 \rightarrow (\lambda x.e) \quad e_2 \rightarrow v}{e_1 \ e_2 \rightarrow v'} \quad e[x \mapsto v] \rightarrow v'}{e_1 \ e_2 \rightarrow v'}$$

$$4. \frac{\frac{\overline{(\lambda x.\lambda y.x) \rightarrow (\lambda x.\lambda y.x)}}{(\lambda x.\lambda y.x) \ 42 \rightarrow \lambda y.42} \quad \frac{\overline{42 \rightarrow 42} \quad \overline{\lambda x.x \rightarrow \lambda x.x}}{(\lambda y.x)[x \mapsto 42] \rightarrow (\lambda y.42)} \quad \overline{x[x \mapsto (\lambda y.42)] \rightarrow (\lambda y.42)}}{(\lambda x.x) \ ((\lambda x.\lambda y.x) \ 42) \rightarrow (\lambda y.42)}$$