

Practice Problems – Type Systems

Sample Solution

1. (a) $\frac{}{\cdot \vdash 42 : int}$
- (b) $\frac{x : int, y : int \vdash y : int}{y : int \vdash \lambda x : int. y : int \rightarrow int}$
- (c) $\frac{\frac{}{y : int, x : int \vdash x : int}}{x : int \vdash \lambda y : int. x : int \rightarrow int}{\cdot \vdash \lambda x : int. \lambda y : int. x : int \rightarrow int \rightarrow int}$
- (d) Let $A = + : int \rightarrow int \rightarrow int$. See Figure 1.
2. There are many possible examples, e.g., $(\lambda x : int \rightarrow int. 42) (\lambda y : int. 1 \ 2)$.
3. (a) (as shown in problem set)

(b)

$$\frac{\frac{y : \beta, x : \alpha \vdash x : \alpha}{x : \alpha \vdash \lambda y. x : \beta \rightarrow \alpha}}{\cdot \vdash \lambda x. \lambda y. x : \alpha \rightarrow \beta \rightarrow \alpha}$$

Solution: α and β are unconstrained

Type: $\alpha \rightarrow \beta \rightarrow \alpha$.

(c)

$$\frac{\frac{\frac{y : \beta, x : \alpha \vdash x : \alpha \quad y : \beta, x : \alpha \vdash y : \beta \quad \alpha = (\beta \rightarrow \gamma)}{y : \beta, x : \alpha \vdash x \ y : \gamma}}{x : \alpha \vdash \lambda y. x \ y : \beta \rightarrow \gamma}}{\cdot \vdash \lambda x. \lambda y. x \ y : \alpha \rightarrow \beta \rightarrow \gamma}$$

Solution: $\alpha = (\beta \rightarrow \gamma)$, and β and γ are unconstrained.

Type: $(\beta \rightarrow \gamma) \rightarrow \beta \rightarrow \gamma$.

(d)

$$\frac{\frac{\frac{y : \beta, x : \alpha \vdash x : \alpha}{x : \alpha \vdash \lambda y. x : \beta \rightarrow \alpha}}{\cdot \vdash (\lambda x. \lambda y. x) : \alpha \rightarrow \beta \rightarrow \alpha} \quad \cdot \vdash 3 : int \quad (\alpha \rightarrow \beta \rightarrow \alpha) = (int \rightarrow \gamma)}{\cdot \vdash (\lambda x. \lambda y. x) \ 3 : \gamma} \\ \frac{\cdot \vdash (\lambda x. \lambda y. x) \ 3 : \gamma \quad \cdot \vdash 42 : int \quad \gamma = (int \rightarrow \delta)}{\cdot \vdash (\lambda x. \lambda y. x) \ 3 \ 42 : \delta}$$

Solution: $\alpha = \beta = \delta = int$, $\gamma = int \rightarrow int$.

Type: int .

$$\begin{array}{c}
\frac{f : int \rightarrow int, A \vdash f : int \rightarrow int}{f : int \rightarrow int, A \vdash f : int \rightarrow int} \quad \frac{f : int \rightarrow int, A \vdash 42 : int}{f : int \rightarrow int, A \vdash f \ 42 : int} \\
\hline
A \vdash (\lambda f.f \ 42) : (int \rightarrow int) \rightarrow int \\
\hline
A \vdash (\lambda f.f \ 42) (\lambda x. + x \ 3) : int
\end{array}
\quad
\begin{array}{c}
\frac{x : int, A \vdash + : int \rightarrow int \rightarrow int}{x : int, A \vdash + x \ 3 : int} \quad \frac{x : int, A \vdash 3 : int}{x : int, A \vdash 3 : int} \\
\hline
x : int, A \vdash + x \ 3 : int \\
\hline
A \vdash (\lambda x. + x \ 3) : int \rightarrow int
\end{array}$$

Figure 1: Derivation for 2.a.iv. The types of function arguments have been omitted for space reasons.