

Lists In Prolog

- ▶ `[a, b, 1, 'hi', [X, 2]]`
- ▶ But really represented as compound terms
 - `[]` is an atom
 - `[a, b, c]` is represented as `.(a, .(b, .(c, [])))`
- ▶ Matching over lists
 - `?- [X, 1, Z] = [a, _, 17]`
 - `X = a,`
 - `Z = 17.`

List Deconstruction

- ▶ Syntactically similar to Ocaml: $[H|T]$ like $h::t$

?- [Head | Tail] = [a,b,c].

Head = a,

Tail = [b, c].

?- [1,2,3,4] = [_, X | _].

X = 2

- ▶ This is sufficient for defining complex predicates

- ▶ Let's define $\text{concat}(L1, L2, C)$

?- $\text{concat}([a,b,c], [d,e,f], X)$.

X = [a,b,c,d,e,f].

Example: Concatenating Lists

- ▶ To program this, we define the “rules” of concatenation
 - If L1 is empty, then $C = L2$
`concat([], L2, L2).`
 - Prepending a new element to L1 prepends it to C, so long as C is the concatenation of L1 with some L2

`concat([E | L1], L2, [E | C]) :-
concat(L1, L2, C).`

- ▶ ... and we're done

Why Is The Return Value An Argument?

- ▶ Now we can ask **what inputs lead to an output**

?- concat(X, Y, [a,b,c]).

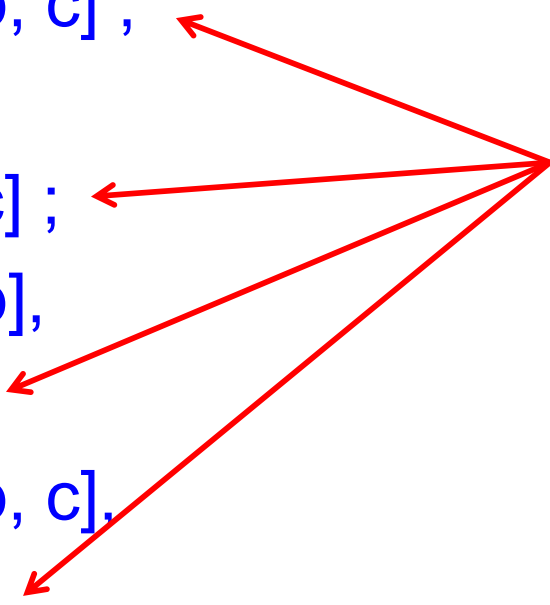
{ X = [],
Y = [a, b, c] ;

{ X = [a],
Y = [b, c] ;

{ X = [a, b],
Y = [c] ;

{ X = [a, b, c],
Y = [] ;

User types ; to request
additional answers



More Syntax: Built-in Predicates

- ▶ Equality (a.k.a. **unification**)
 $X = Y$ $f(1,X,2) = f(Y,3,_)$
- ▶ **fail** and **true**
- ▶ “Consulting” (loading) programs
`?- consult('file.pl')` `?- ['file.pl']`
- ▶ Output/Input
`?- write('Hello world'), nl` `?- read(X).`
- ▶ (Dynamic) type checking
`?- atom(elephant)` `?- atom(Elephant)`
- ▶ **help**

The == Operator

- ▶ For identity comparisons
- ▶ $X == Y$
 - Returns true if and only if X and Y are identical

- ▶ Examples

?- 9 == 9.

true.

?- X == 9.

False.

?- X == X.

true.

?- 9 == 7+2.

false.

?- X == Y.

false.

?- 7+2 == 7+2.

true.

The ::= Operator

- ▶ For arithmetic operations
- ▶ “LHS ::= RHS”
 - Evaluate the LHS to value V1 (Error if not possible)
 - Evaluate the RHS to value V2 (Error if not possible)
 - Then match: $V1 = V2$

- ▶ Examples

?- 7+2 ::= 9.
true.

?- 7+2 ::= 3+6.
true.

?- X ::= 9.

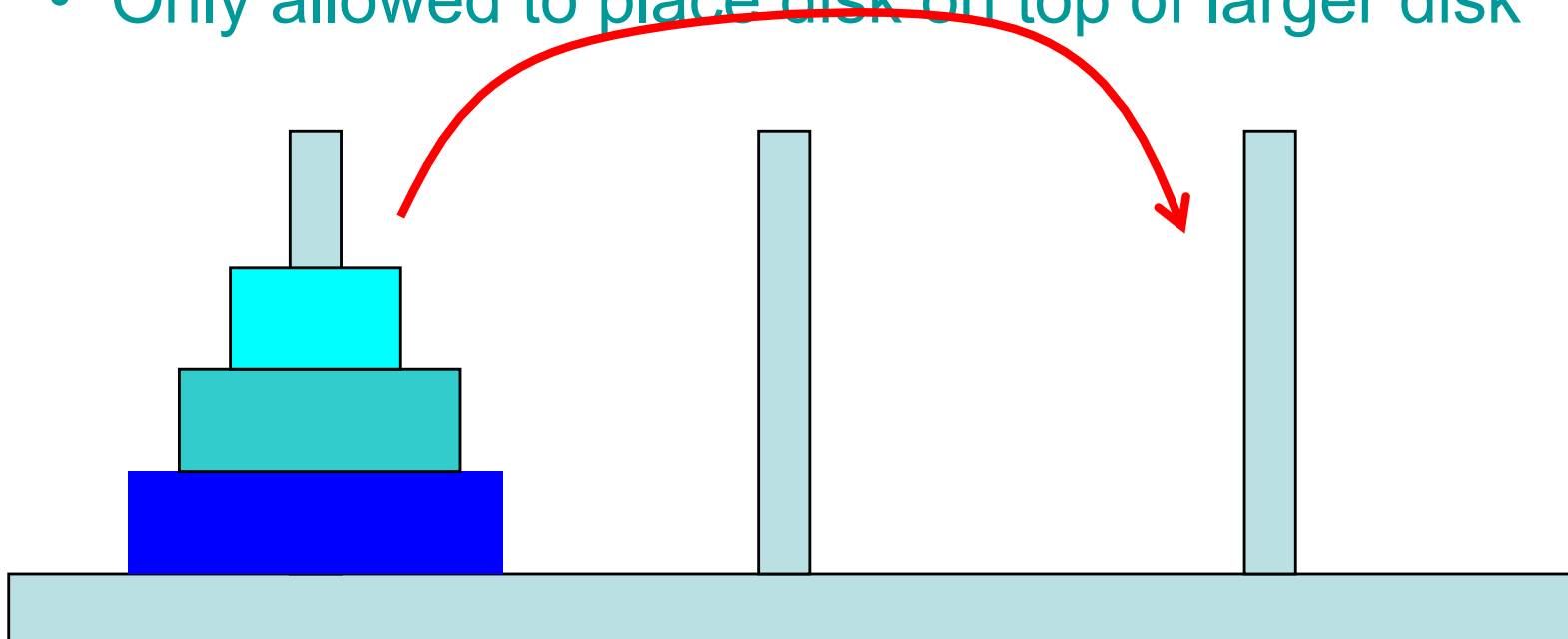
?- X ::= 7+2

Error: =:/2: Arguments are not sufficiently instantiated

Example – Towers of Hanoi

► Problem

- Move stack of disks between pegs
- Can only move top disk in stack
- Only allowed to place disk on top of larger disk



Example – Towers of Hanoi

- ▶ To move a stack of n disks from peg X to Y
 - Base case
 - If $n = 1$, move disk from X to Y
 - Recursive step
 1. Move top $n-1$ disks from X to 3rd peg
 2. Move bottom disk from X to Y
 3. Move top $n-1$ disks from 3rd peg to Y

Iterative algorithm would take much longer to describe!

Towers of Hanoi

► Code

```
move(1,X,Y,_) :-
```

```
    write('Move top disk from '), write(X),  
    write(' to '), write(Y), nl.
```

```
move(N,X,Y,Z) :-
```

```
    N>1,
```

```
    M is N-1,
```

```
    move(M,X,Z,Y),
```

```
    move(1,X,Y,_),
```

```
    move(M,Z,Y,X).
```

Prolog Terminology

- ▶ A query, goal, or term where variables do not occur is called **ground**; else it's **nonground**
 - $\text{foo}(a,b)$ is ground; $\text{bar}(X)$ is nonground
- ▶ A **substitution** θ is a partial map from variables to terms where $\text{domain}(\theta) \cap \text{range}(\theta) = \emptyset$
 - Variables are terms, so a substitution can map variables to other variables, but not to themselves
- ▶ A is an **instance** of B if there is a substitution such that $A = B\theta$ ← The substitution θ applied to B
- ▶ C is a **common instance** of A and B if it is an instance of A and an instance of B

Prolog's Algorithm Solve()

Starts as empty

Solve(goal G , program P , substitution θ) =

- ▶ Suppose G is A_1, \dots, A_n . Choose goal A_1 .
- ▶ For each clause $A :- B_1, B_2, \dots, B_k$ in P ,
 - if θ_1 is the **mgu** of A and $A_1\theta$ then
 - ▶ If **Solve**($\{B_1, \dots, B_k, A_2, \dots, A_n\}, P, \theta \cdot \theta_1$) = some θ' then **return** θ'
 - ▶ (else it has failed, so we continue the for loop)
 - (else unification has failed, so try another rule)
- ▶ If loop exits return **fail**
- ▶ **Output**: θ s.t. $G\theta$ can be deduced from P , or fail

Chooses goals in order

Most
General
Unifier

Implements backtracking

! : a.k.a. “cut”

- ▶ When a ! is reached, it succeeds and commits Prolog to all the choices made since the parent goal was unified with the head of the clause the cut occurs in
 - Suppose we have clause C which is $A :- B_1, \dots, B_k, !, \dots, B_n$.
 - If the current goal unifies with A, and B_1, \dots, B_k further succeed, the program is committed to the choice of C for the goal.
 - If any B_i for $i > k$ fail, backtracking only goes as far as the cut.
 - If the cut is reached when backtracking, **the goal fails**

Cut

- ▶ Limits backtracking to predicates to **right** of cut

- ▶ Example

jedi(luke).

jedi(yoda).

sith(vader).

sith(maul).

fight2(X,Y) :- jedi(X), **!**, sith(Y).

fight3(X,Y) :- jedi(X), sith(Y), **!**.

?- fight2(A,B).

A=luke,

B=vader;

A=luke,

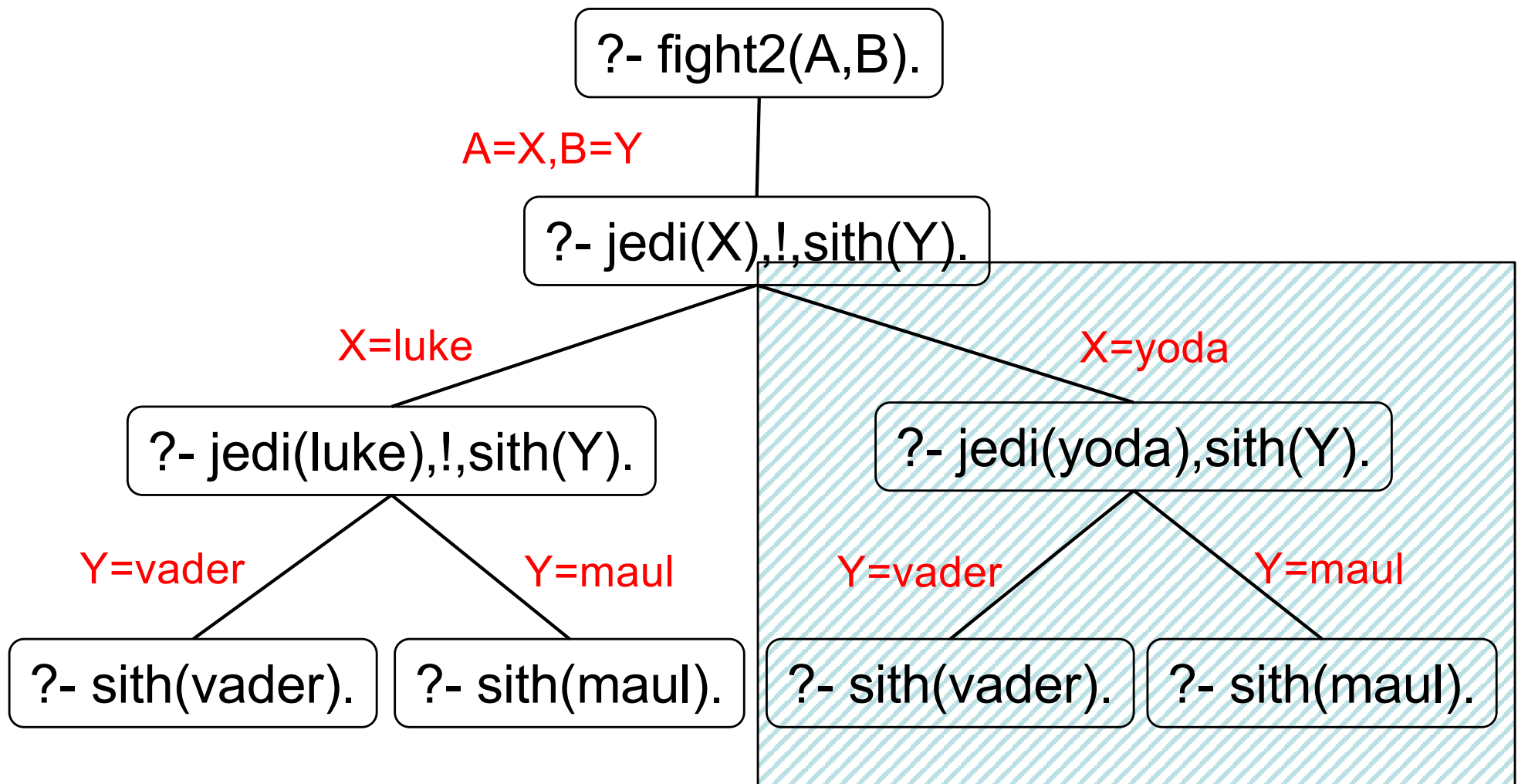
B=maul.

?- fight3(A,B).

A=luke,

B=vader.

Prolog Search Tree Limited By Cut



What Exactly Is Cut Doing?

Prunes all clauses below it

Prunes alternative solutions to its left

Does *not* affect the goals to its right

Note: Cut only affects **this** call to merge. Does not affect backtracking of functions calling merge, or later recursive call to merge past cut

```
merge([X|Xs], [Y|Ys], [X|Zs]) :-  
X < Y, !, merge(Xs, [Y|Ys], Zs).
```

```
merge([X|Xs], [Y|Ys], [X,Y|Zs]) :-  
X == Y, !, merge(Xs, Ys, Zs).
```

```
merge([X|Xs], [Y|Ys], [Y|Zs]) :-  
X > Y, !, merge([X|Xs], Ys, Zs).
```

```
merge(Xs, [], Xs) :- !.
```

```
merge([], Ys, Ys) :- !.
```


Why Use Cuts?

- ▶ Save time and space, or eliminate redundancy
 - Prune useless branches in the search tree
 - If sure these branches will not lead to solutions
 - These are **green cuts**

- ▶ Guide to the search to a different solution
 - Change the meaning of the program
 - Intentionally returning only subset of possible solutions
 - These are **red cuts**

Negation As Failure

- ▶ Cut may be used implement negation (not)
- ▶ Example
 - `not(X) :- call(X), !, fail.`
 - `not(X).`
- ▶ If `X` succeeds, then the cut is reached, committing it; `fail` causes the whole thing to fail
- ▶ If `X` fails, then the second rule is reached, and the overall goal succeeds.
 - FYI, `X` here refers to an arbitrary goal
 - Effect of `not` depends crucially on rule order

Not

- ▶ Not is tricky to use
 - Does not mean “not true”
 - Just means “not provable at this time”

- ▶ Example

jedi(luke).

jedi(vader).

sith(vader).

Cannot prove either
jedi(leia) or sith(leia)
are true, so not()
returns true

?- not(sith(luke)).

true.

?- not(sith(vader)).

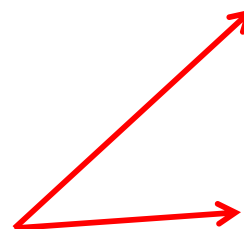
false.

?- not(jedi(leia)).

true.

?- not(sith(leia)).

true.



Not (cont.)

▶ Not is tricky to use

- Does not mean “not true”
- Just means “not provable at this time”

?- not(sith(X)).

false.



Huh? Why not return X=luke?

▶ Example

jedi(luke).

jedi(vader).

sith(vader).

Because not(sith(X)) does not mean
“Can prove sith(X) is false for some X”

not(sith(X)) :- sith(X), !, fail.
not(sith(X)).

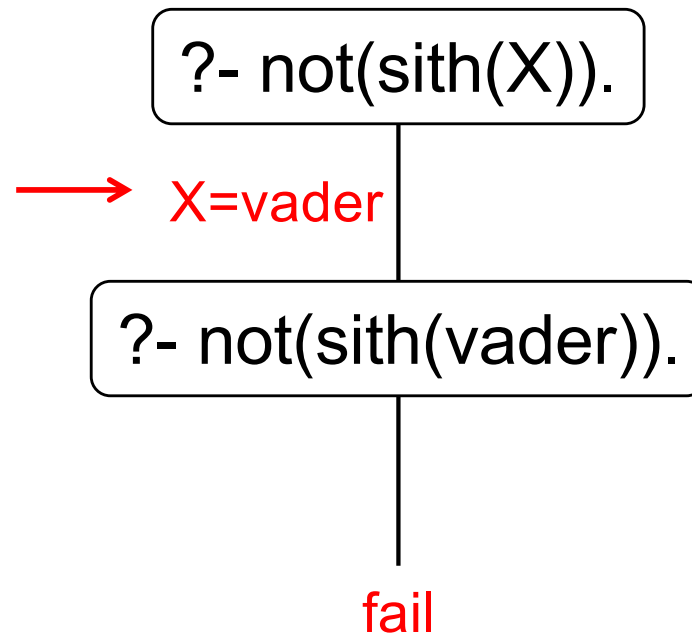
Instead, it means “Cannot prove sith(X) is true for some X”. So X=vader causes not(sith(X)) to fail and return false

Not – Search Tree

jedi(luke).
jedi(vader).
sith(vader).

not(sith(X)) :- sith(X), !, fail.
not(sith(X)).

Will search for
all X such that
sith(X) is true.



Not (cont.)

- ▶ Ordering of clauses matters

- ▶ Example

jedi(luke).

jedi(vader).

sith(vader).

true_jedi1(X) :-

 jedi(X), not(sith(X)).

true_jedi2(X) :-

 not(sith(X)), jedi(X).

?- true_jedi1(luke).

true.

?- true_jedi1(X).

X=luke.

?- true_jedi2(luke).

true.

?- true_jedi2(X).

false.



X=vader causes not(sith(X)) to fail;
Will not backtrack to X=luke, since
sith(luke) is not a fact

True_jedi2 – Search Tree

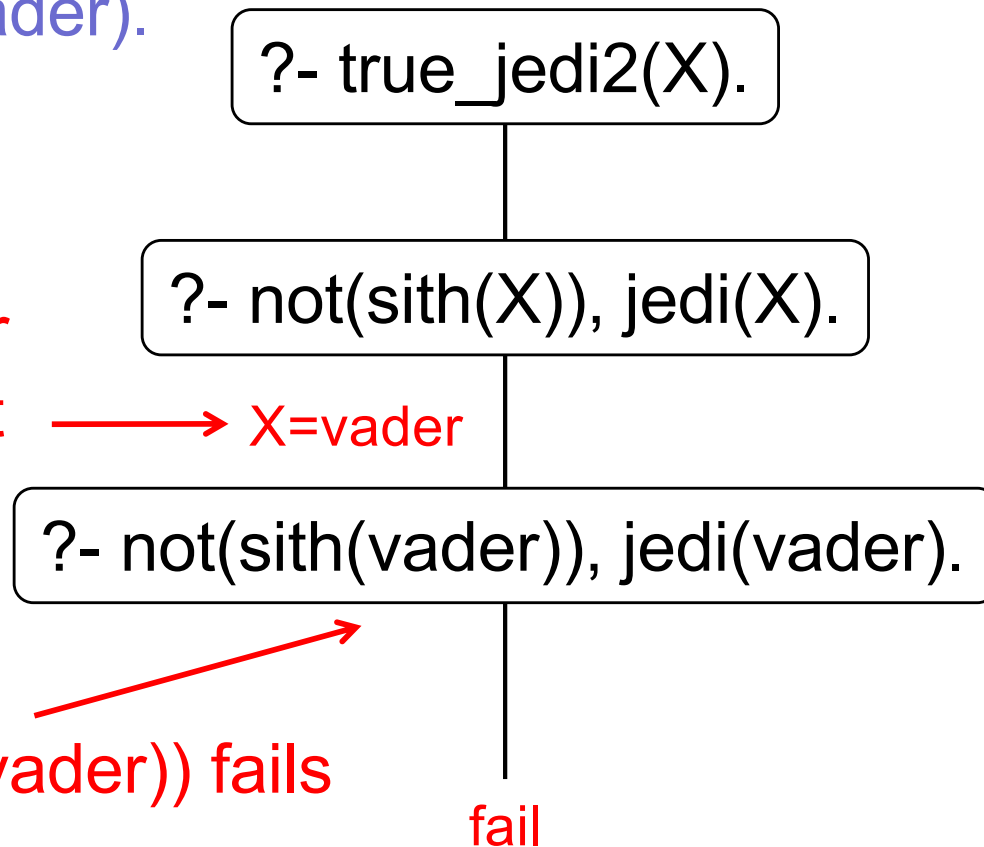
jedi(luke).
jedi(luke).
sith(vader).

not(sith(X)) :- sith(X), !, fail.
not(sith(X)).

Will search for
all X such that
sith(X) is true.

→ X=vader

not(sith(vader)) fails



Not and \=

- ▶ Built-in operators
 - \+ is not
 - $X \neq Y$ is same as $\text{not}(X=Y)$
 - $X \neq\equiv Y$ is same as $\text{not}(X\equiv Y)$
- ▶ So be careful using \=
 - Ordering of clauses matters
 - Try to ensure operands of \= are instantiated

Example Using \neq

► Example

jedi(luke).

jedi(yoda).

help2(X,Y) :- jedi(X), jedi(Y), X \neq Y.

help3(X,Y) :- jedi(X), X \neq Y, jedi(Y).

help4(X,Y) :- X \neq Y, jedi(X), jedi(Y).

?- help2(X,Y).

X=luke,

Y=yoda;

X=yoda,

Y=luke.


?- help3(X,luke).

X=yoda.

?- help3(X,Y).

false.

After selecting X,
can choose Y=X
and fail X \neq Y.



Help3 – Search Tree

`not(X=Y) :- X=Y, !, fail.`
`not(X=Y).`

`jedi(luke).`
`jedi(yoda).`

`?- help3(X,Y).`

`help3(X,Y) :-`
`jedi(X),`
`X \= Y,`
`jedi(Y).`

`?- jedi(X), X \= Y, jedi(Y).`

`X=luke`

`X=yoda`

`?- jedi(luke), luke \= Y, jedi(Y).`

`?- jedi(yoda), yoda \= Y, jedi(Y).`

`Y=luke`

`Y=yoda`

`?- luke \= luke`

`?- yoda \= yoda`

`luke=luke,! ,fail`

`yoda=yoda,! ,fail`

Using \neq

- ▶ In fact, given $X \neq Y$
 - will always fail if X or Y are not both instantiated

$X \neq a$ // fails for $X=a$

$a \neq Y$ // fails for $Y=a$

$X \neq Y$ // fails for $X=Y$

Example Using \neq

► Example

jedi(luke).

jedi(yoda).

help2(X,Y) :- jedi(X), jedi(Y), $X \neq Y$.

help3(X,Y) :- jedi(X), $X \neq Y$, jedi(Y).

help4(X,Y) :- $X \neq Y$, jedi(X), jedi(Y).

?- help4(X,luke).

false.

?- help4(yoda,luke).

true.

Built-in List Predicates

- ▶ `length(List,Length)`
 - ?- `length([a, b, [1,2,3]], Length).`
`Length = 3.`
- ▶ `member(Elem,List)`
 - ?- `member(duey, [huey, duey, luey]).`
`true.`
 - ?- `member(X, [huey, duey, luey]).`
`X = huey; X = duey; X = luey.`
- ▶ `append(List1,List2,Result)`
 - ?- `append([duey], [huey, duey, luey], X).`
`X = [duey, huey, duey, luey].`

Built-in Predicates

- ▶ `sort(List, SortedList)`

 - ?- `sort([2,1,3], R).`

 - `R= [1,2,3].`

- ▶ `findall(Elem, Predicate, ResultList)`

 - ?- `findall(E, member(E, [huey, duey, luey]), R).`

 - `R=[huey, duey, luey].`

- ▶ `setof(Elem, Predicate, ResultSortedList)`

 - ?- `setof(E, member(E, [huey, duey, luey]), R).`

 - `R=[duey, huey, luey].`

- ▶ See documentation for more

 - <http://www.swi-prolog.org/pldoc/man?section=builtin>

Prolog Summary

- ▶ General purpose logic programming language
 - Associated with AI, computational linguistics
 - Also used for theorem proving, expert systems
- ▶ Declarative programming
 - Specify facts & relationships between facts (rules)
 - Run program as queries over these specifications
- ▶ Natural support for
 - Searching within set of constraints
 - Backtracking