Turn in Section 6 of the NP-completeness homework separately.

1. Consider an undirected, graph on four vertices: (a, b), (b, c), (c, d), and (d, a). This is a cycle of size four. It has exactly two Eulerian cycles: a, b, c, d and a, d, c, b. Rotations of those two cycles are considered the same cycle.
   (a) Give an undirected graph that has exactly four Eulerian cycles.
   (b) Show that there is an undirected graph with exactly 2^k Eulerian cycles for any integer $k \geq 1$.
   (c) Give an example of an undirected graph with some number of Eulerian cycles that is not a power of 2. How many Eulerian cycles does your graph have? Justify.
   (d) **Challenge problem.** For what values of s can an undirected graph have exactly s Eulerian cycles? Justify.

2. Show that for every (natural number) t, there exists a weighted, undirected graph $G = (V, E)$ with source vertex s, where all edge weights are distinct, such that G has exactly t different shortest path trees.

3. Let $G = (V, E)$ be directed, weighted graph with exactly one negative weight edge, but no negative weight cycles. Give an efficient algorithm to solve the single source shortest path problem. Analyze it execution time. Informally justify the correctness of your algorithm.