Turn in Section 3 of the NP-completeness homework separately.

1. Assume that you guess that

$$\sum_{k=1}^{n} k^2 = an^3 + bn^2 + cn + d \; .$$

Use constructive induction to find the constants a, b, c, and d and prove that your guess was correct.

2. Do not use integrals for this problem. Do not worry about floors and ceilings (so you may assume that n is "nice"). Ignore second order terms. Consider

$$\sum_{k=1}^{n} k^2 \; .$$

   (a) Split the sum into two equal-sized regions to obtain an upper bound for its value.
   (b) Split the sum into two equal-sized regions to obtain a lower bound for its value (as done in class).
   (c) Show how to obtain a better upper bound by splitting the sum into two unequal-sized regions. Make your bound as tight as possible. How does your bound compare with the upper bound obtained in Part (a)?
   (d) Show how to obtain a better lower bound by splitting the sum into two unequal-sized regions. Make your bound as tight as possible. How does your bound compare with the lower bound obtained in Part (b)?

3. (a) Use the integral method to obtain upper and lower bound bounds for

$$\sum_{k=1}^{n} k^2 \; .$$

   (b) How do your bounds compare with those obtained in Problem 2?
   (c) How do your bounds compare with the exact polynomial in Problem 1?

4. Consider an array of size eight with the numbers $80, 30, 40, 70, 10, 20, 60, 50$. Assume you execute quicksort using the version of partition from CLRS. Note that in this algorithm an element might exchange with itself (which counts as one exchange).

   (a) Show the array after the first partition. How many comparisons and exchanges are used?
   (b) Show the left side (of the original pivot) after the next partition. How many comparisons are used? How many exchanges?
   (c) Show the right side (of the original pivot) after the next partition on that side. How many comparisons are used? How many exchanges?