Put your *name* and *section number* on your solution and (if more than one sheet) staple.
Section 4 of the NP-completeness homework will be due Friday, October 21.

1. Assume that your machine has an instruction that does two comparisons in one comparison step. The syntax might look something like this:

```
compare A with B and C with D
    case A<B and C<D: ... exit
    case A<B and C≥D: ... exit
    case A≥B and C<D: ... exit
    case A≥B and C≥D: ... exit
end compare
```

   The variables A,B,C,D do not have to be distinct.

   (a) Give an efficient algorithm for partition using this special comparison operation. Try to minimize the number of comparison steps. Write the pseudo-code. For simplicity, you may assume that the length of the list is odd. How many comparison steps does your partition algorithm use on a list of size n?

   (b) Give an efficient algorithm to partition a list into three groups – small, medium, and large – using this special comparison operation. There are at least a couple ways to do this. Your algorithm should pivot on the last two elements, and, at each comparison step, compare both of them to the same element in the list. (This is a little bit wasteful and inefficient, but it is your algorithm.) Write the pseudo-code. Return the two pivot postions in order as a pair. How many comparison steps does your partition algorithm use on a list of size n?

   (c) Assume that your algorithm in Part (b) always partitions the elements into three equal-sized groups during every partition in Quick Sort. Write a recurrence for (an upper bound on) the number of comparison steps in Quick Sort. Do not worry about floors and ceilings.

   (d) Solve your recurrence from Part (c) using constructive induction. Just get the high order term. (Simplify your answer.)

2. Consider the function

$$5n^3 + 3n(\lg n)^2 + 2n^2 + 6n$$

   (a) What is the high order term?

   (b) What is the second order term?

   (c) What is the third order term?

   (d) Write the function in Theta notation (in the simplest way).

3. For each pair of expressions (A, B) below, indicate whether A is O, o, Ω, ω, or Θ of B. Note that zero, one or more of these relations may hold for a given pair; list all correct ones.

	A	B
(a)	$8n^3 + 14n^2 + 5$	$n^4/5$
(b)	2^n	n^{50}
(c)	$5\sqrt{n}$	$(\log n)^{12}$
(d)	$\sqrt[3]{n}$	$n^{\cos(\pi n/8)}$
(e)	25^n	5^{2n}
(f)	$2^{(n \lg n)}$	10^n
(g)	$\log(n!)$	$n \log n$