Asymptotic Notations.

$\Theta(g(n)) = \{f(n) : \exists c_1, c_2, n_0 > 0 \text{ s.t. } 0 \le c_1 g(n) \le f(n) \le c_2 g(n) \text{ for all } n \ge n_0\}.$

$O(g(n)) = \{f(n) : \exists c, n_0 > 0 \text{ s.t. } 0 \le f(n) \le c g(n) \text{ for all } n \ge n_0\}.$

$\Omega(g(n)) = \{f(n) : \exists c, n_0 > 0 \text{ s.t. } 0 \le c g(n) \le f(n) \text{ for all } n \ge n_0\}.$

$f(n) = o(g(n))$ if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$.

$f(n) = \omega(g(n))$ if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$.

$f(n) \sim g(n)$ if $f(n) = g(n) + o(g(n))$.

Logarithms.

$$a = b^{\log_b a} \qquad \log_c(ab) = \log_c a + \log_c b \qquad \log_b a^n = n \log_b a$$

$$\log_b a = \frac{\log_c a}{\log_c b} \qquad \log_b(1/a) = -\log_b a \qquad \log_b a = \frac{1}{\log_a b} \qquad a^{\log_b n} = n^{\log_b a}$$

Stirling's Formula.

$$n! \;\approx\; \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$$

Probability.

$$\mathrm{E}[X] = \sum_x x \Pr\{X = x\}\ , \qquad \mathrm{Var}[X] = \mathrm{E}[(X - \mathrm{E}(X))^2] = \mathrm{E}[X^2] - \mathrm{E}^2[X]\ , \qquad \sigma[X] = \sqrt{\mathrm{Var}[X]}\ .$$

Quadratic Formula.

$$ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Approximation by integrals:

$$\int_{m-1}^{n} f(x)dx \le \sum_{k=m}^{n} f(k) \le \int_{m}^{n+1} f(x)dx \qquad \text{for } f(x) \text{ monotonically increasing}$$

$$\int_{m}^{n+1} f(x)dx \le \sum_{k=m}^{n} f(k) \le \int_{m-1}^{n} f(x)dx \qquad \text{for } f(x) \text{ monotonically decreasing}$$

Summations.

Distribution law:

$$\left(\sum_{i=1}^{m} a_i\right)\left(\sum_{j=1}^{n} b_j\right) = \sum_{i=1}^{m}\left(\sum_{j=1}^{n} a_i b_j\right)$$

Interchanging order of summation:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} = \sum_{j=1}^{n}\sum_{i=1}^{m} a_{ij}$$

Splitting range:

$$\sum_{k=1}^{n} a_k = \sum_{k=1}^{r} a_k + \sum_{k=r+1}^{n} a_k$$

Arithmetic series:

$$\sum_{k=1}^{n} k = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

Quadratic series:

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k(k+1) = 1\cdot 2 + 2\cdot 2 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

Geometric series:

$$\sum_{k=0}^{n} x^k = 1 + x + +x^2 \cdots + x^n = \frac{x^{n+1}-1}{x-1} \qquad x \neq 1$$

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \qquad |x| < 1$$

Harmonic series:

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} = \sum_{k=1}^{n} \frac{1}{k} = \ln n + O(1)$$

Telescoping series:

$$\sum_{k=1}^{n} (a_k - a_{k-1}) = a_n - a_0$$

Products.

$$\prod_{k=1}^{n} a_k = a_1 a_2 \cdots a_n \qquad \log \prod_{k=1}^{n} a_k = \sum_{k=1}^{n} \log a_k$$