School Choice
and the Boston Mechanism

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School Choice Problem

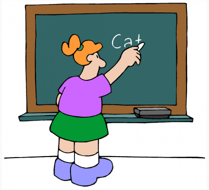
Students
$I = \{i_1, i_2, \dots, i_n\}$

Preference ($\succ_i$) over schools

$\mathbf{1}: \{a \succ b \succ c\}$

$\mathbf{2}: \{b \succ a \succ c\}$

$\mathbf{3}: \{a \succ b \succ c\}$



Schools
$S = \{s_1, s_2, \dots, s_m\}$

Priority over students

Capacity

$\mathbf{a}: \{2, 1, 3\} \mid (\ )$

$\mathbf{b}: \{3, 1, 2\} \mid (\ )$

$\mathbf{c}: \{3, 2, 1\} \mid (\ )(\ )$

College Admission Problem: Both schools and students are strategic agents (two-sided matching)

School Choice Problem: Only student welfare matters (one-sided matching)

- Schools **do not choose** priorities
- Similar to resource allocation

Boston Mechanism

1954: Brown v. Board of Education required desegregation of schools.

1974: In Boston, a court order forced racial integration of public schools via, leading to protests.

1981: Preempting a court order, Cambridge introduced the the **controlled choice program** to...

- Increase diversity
- Treat students fairly
- Give families a choice

The **Boston Mechanism** was adopted by many US school districts.

Boston Mechanism

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1974: In Boston, a court order forced racial integration of public schools via, leading to protests.

1981: Preempting a court order, Cambridge introduced the the **controlled choice program** to...

- Increase diversity
- Treat students fairly
- Give families a choice

The **Boston Mechanism** was adopted by many US school districts.

Boston Mechanism:

Assign as many students as possible to their first choice of school.

Round 1: Only consider each student's **first choice**: For each school, assign students to seats based on priority.

⋮ ⋮

Round k: Consider each student's **k^{th} choice**: For each school, assign students to seats based on priority.

Boston Mechanism

What can we say about this mechanism?

- **Pareto Efficient**
- **Individually Rational** (if every student prefers being assigned to being unassigned)
- Strategy-proof?...

Boston Mechanism:

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Boston Mechanism

Some advice for strategic families[1]:

> *"For a better chance of your 'first choice' school . . . consider choosing less popular schools."* (BPS School Guide)
>
> *"... find a school you like that is undersubscribed and put it as a top choice, OR, find a school that you like that is popular and put it as a first choice and find a school that is less popular for a 'safe' second choice."* (West Zone Parent Group)

Let's consider the **preference revelation game** induced by the Boston Mechanism

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1: Abdulkadiroglu, Atila. "School choice." *The Handbook of Market Design*(2013).

Boston Game

Students

$I = \{i_1, i_2, \ldots, i_n\}$

Schools

$S = \{s_1, s_2, \ldots, s_m\}$

1 : $\{ a > b > c \}$

2 : $\{ b > a > c \}$

3 : $\{ a > b > c \}$

a : $\{ 2, 1, 3 \}$ | ( )

b : $\{ 3, 1, 2 \}$ | ( )

c : $\{ 3, 2, 1 \}$ | ( ) ( )

Boston Game

Students
$I = \{i_1, i_2, \dots, i_n\}$

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1 : { $a \succ b \succ c$ } → a : { 2, 1, 3 } | (1)

2 : { $b \succ a \succ c$ } → b : { 3, 1, 2} | (2)

3 : { $a \succ b \succ c$ }

c : { 3, 2, 1} | ( ) ( )

Round 1:
- **1**,**3** propose to a (only **1** is accepted)
- **2** proposes to **b** (accepted)

Boston Game

Students

$I = \{i_1, i_2, \dots, i_n\}$

Schools

$S = \{s_1, s_2, \dots, s_m\}$

$\mathbf{1} : \{ \boldsymbol{a} \succ b \succ c \}$ → $\mathbf{a} : \{ 2, \mathbf{1}, 3 \}$ | ($\mathbf{1}$)

$\mathbf{2} : \{ \boldsymbol{b} \succ a \succ c \}$ → $\mathbf{b} : \{ 3, 1, \mathbf{2} \}$ | ($\mathbf{2}$)

$\mathbf{3} : \{ a \succ b \succ c \}$ $\qquad$ $\mathbf{c} : \{ 3, 2, 1 \}$ | ( ) ( )

Round 1:

- **1,3** propose to a (only **1** is accepted)
- **2** proposes to **b** (accepted)

Round 2:

- **3** proposes to **b** (rejected)

Boston Game

Students
$I = \{i_1, i_2, \dots, i_n\}$

Schools
$S = \{s_1, s_2, \dots, s_m\}$

1 : { ***a*** > *b* > *c* } → **a** : { 2, **1**, 3 } | (**1**)

2 : { ***b*** > *a* > *c* } → **b** : { 3, 1, **2**} | (**2**)

3 : { *a* > *b* > ***c*** } → **c** : { 3, 2, 1} | (**3**) ( )

Round 1:

- **1**,**3** propose to a (only **1** is accepted)
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Round 2:

- **3** proposes to **b** (rejected)

Round 3:

- **3** proposes to **c** (accepted)

Is this a stable matching?

- Blocking pair: (**3**,**b**)
- **3** can improve assignment by playing { *b* > … }

Can we generalize?

Boston Game

Preferences and $\succ_i$ and priorities P define an **Economy**

1 : { $a \succ b \succ c$ }	**a** : { 2, 1, 3 } | ( )
2 : { $b \succ a \succ c$ }	**b** : { 3, 1, 2} | ( )
⋮	⋮
i_n : { $a \succ b \succ c$ }	s_m : { 3, 2, 1} | ( ) ( )

The set of Nash equilibria of the Boston game is the set of stable matchings under the two-sided matching problem. (Ergin & Sonmez, 2006).

Intuition:

- If (i_i, s_j) is a blocking pair, student i_i can guarantee a seat in school s_j by ranking it as her **top choice.**

Any stable matching μ can be selected if all students request their assignment under this matching $\mu(i)$ as their **top choice**.

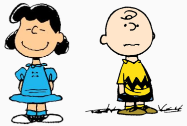
What if some students don't strategize?

Sincere and Strategic Students

What if some students don't strategize?

Strategic Students

Sincere Students



I_1^i: Sincere students who rank school i as first choice under **and all strategic students**

I_2^i: Sincere students who rank school i as second choice

⋮

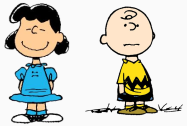
I_n^i: Sincere students who rank school i as n^{th} choice

Sincere and Strategic Students

Augmented Economy

$1 : \{ a > b > c \}$	$a : \{ I_1^a, I_2^a, \ldots, I_3^a, 2, 1, 3 \}$
$2 : \{ b > a > c \}$	$b : \{ I_1^b, I_2^b, \ldots, I_3^b, 3, 1, 2 \}$
⋮	⋮
$I_n : \{ a > b > c \}$	$s_m : \{ I_1^m, I_2^m, \ldots, I_3^m, 3, 2, 1 \}$

WIth sincere and strategic students, the Nash equilibria of the Boston game is the set of stable matchings under the augmented economy. (Pathak & Sonmez, 2008).



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I_n^i: Sincere students who rank school i as n^th choice

Sincere and Strategic Students

Augmented Economy

$\mathbf{1}: \{a \succ b \succ c\}$	$\mathbf{a}: \{I_1^a, I_2^a, \ldots, I_3^a, 2, 1, 3\}$
$\mathbf{2}: \{b \succ a \succ c\}$	$\mathbf{b}: \{I_1^b, I_2^b, \ldots, I_3^b, 3, 1, 2\}$
$\vdots$	$\vdots$
$I_n: \{a \succ b \succ c\}$	$s_m: \{I_1^m, I_2^m, \ldots, I_3^m, 3, 2, 1\}$

WIth sincere and strategic students, the Nash equilibria of the Boston game is the set of stable matchings under the augmented economy. (Pathak & Sonmez, 2008).

Intuition:

- For every school ***s***, students who rank ***s*** higher than other students *effectively* have higher priority at ***s***.

(Def'n:) Strategic students select the **Pareto-dominant Nash equilibrium**

Implications:

- Sincere students' assignments are the same under every Nash
- If a sincere student ***I*** *becomes strategic:*
  - Student ***I*** weakly benefits
  - All other strategic students weakly suffer

Can we do better?

Can we do better?

Yes! With the **student-optimal stable matching** (Gale & Shapley)

Boston Game	**Student-Optimal Mechanism**
Leads to...	Leads to...
Pareto-dominant Nash equilibrium	**Student-optimal stable matching**
...which is preferred by	...which is preferred by
- **All strategic** - **Some sincere**	**All students**

Deferred Acceptance Mechanism:

Find the student-optimal stable matching

Round 1: Students propose their 1^{st} **choice**. Seats are *tentatively* accepted based on priority and capacity. Unassigned students are rejected.

⋮ ⋮

Round k: Unassigned students propose to their **next** choice. Full schools accept if proposers have higher priority than current students and reject otherwise.

Deferred Acceptance

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Round 1:

- **1** proposes to **a** (accepted)
- **2** proposes to **b** (accepted)
- **3** proposes to **a** (rejected)

Deferred Acceptance

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3 : $\{ a > b > c \}$ → **b**; **c** : { 3, 2, 1} | ( ) ( )

Round 1:

- **1** proposes to **a** (accepted)
- **2** proposes to **b** (accepted)
- **3** proposes to **a** (rejected)

Round 2:

- **3** proposes to **b** (accepted)
  - **2** is rejected

Deferred Acceptance

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Schools
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$\mathbf{3} : \{ a \succ b \succ c \}$

$\mathbf{a} : \{ 2, 1, 3 \} \mid (2)$

$\mathbf{b} : \{ 3, 1, 2 \} \mid (3)$

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Round 1:
- **1** proposes to **a** (accepted)
- **2** proposes to **b** (accepted)
- **3** proposes to **a** (rejected)

Round 2:
- **3** proposes to **b** (accepted)
  - **2** is rejected

Round 3:
- **2** proposes to **a** (accepted)
  - **1** is rejected

Deferred Acceptance

Students

$I = \{i_1, i_2, \dots, i_n\}$

Schools

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$1 : \{ a \succ b \succ c \}$

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$3 : \{ a \succ b \succ c \}$

$a : \{ 2, 1, 3 \} \mid (2)$

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Round 1:

- **1** proposes to **a** (accepted)
- **2** proposes to **b** (accepted)
- **3** proposes to **a** (rejected)

Round 2:

- **3** proposes to **b** (accepted)
  - **2** is rejected

Round 3:

- **2** proposes to **a** (accepted)
  - **1** is rejected

Round 3,4:

- **1** proposes to **b** (rejected)
- **1** proposes to **c** (accepted)

Deferred Acceptance

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a : { 2, 1, 3 } | (2)

b : { 3, 1, 2} | (3)

c : { 3, 2, 1} | (1) ( )

Deferred Acceptance:
- Preferred by **2, 3**

Boston Mechanism:
- Preferred by **1**

Deferred acceptance is preferred by *most* non-strategic students…

But how much does this matter?

So what?

The core is small…

Table 1—Average Number of Students Receiving Different Schools in Student-Optimal versus School-Optimal Matching

	Fraction of sincere students			
	20 percent	40 percent	60 percent	80 percent
2005–2006				
Grade K2	0.14	0.08	0.04	0.01
Grade 6	0.38	0.20	0.07	0.01
2006–2007				
Grade K2	0.03	0.01	0.00	0.00
Grade 6	0.24	0.14	0.05	0.01

Note: This table is based on data provided by Boston Public Schools for Round 1 of their admissions process in 2005–2006 and 2006–2007.

So what?

Several reasons to use prefer a **strategy-proof mechanism**

- Strategies can be wrong (**inefficient**)
- Schools want *true* preference data
- Students waste time strategizing
- Boston Mechanism doesn't guarantee a stable matching

But what if students *continue to strategize* under a strategy-proof mechanism?

So what?

The transition to strategy-proofness won't rock the boat.

Table 14— 2001-2002 Naive Comparison of Mechanisms[a]

Stated Choice	Boston Mechanism		Student-proposing Deferred Acceptance		Top Trading Cycles Mechanism	
	number	percent	number	percent	number	percent
Panel A: Elementary school applicants						
1st	2,590	77.9	2,451	73.7	2,464	74.1
2nd	309	9.3	419	12.6	410	12.3
3rd	103	3.1	173	5.2	171	5.1
4th	23	0.7	55	1.7	55	1.7
5th	12	0.4	23	0.7	23	0.7
Unassigned	289	8.7	205	6.2	203	6.1
Panel B: Middle school applicants						
1st	4,197	77.3	3,922	72.2	3,938	72.5
2nd	417	7.7	701	12.9	689	12.7
3rd	269	5.0	328	6.0	317	5.8
4th	44	0.8	75	1.4	68	1.3
5th	17	0.3	23	0.4	23	0.4
Unassigned	485	8.9	380	7.0	394	7.3
Panel C: High school applicants						
1st	5,486	86.0	5,261	82.5	5,328	83.5
2nd	407	6.4	624	9.8	587	9.2
3rd	158	2.5	236	3.7	202	3.2
4th	38	0.6	36	0.6	31	0.5
5th	6	0.1	7	0.1	6	0.1
Unassigned	285	4.5	216	3.4	226	3.5

Thank you!