

# APPLIED MECHANISM DESIGN FOR SOCIAL GOOD

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Lecture #6 – 09/15/2016

**CMSC828M**  
**Tuesdays & Thursdays**  
**12:30pm – 1:45pm**



**COMPUTER SCIENCE**  
UNIVERSITY OF MARYLAND

**SCRIBE LIST IS ONLINE!**  
*(ALSO A SUGGESTED LATEX TEMPLATE)*

# **THIS CLASS: STACKELBERG & SECURITY GAMES**

# SIMULTANEOUS PLAY

Previously, assumed players would play **simultaneously**

- Two drivers simultaneously decide to go straight or divert
- Two prisoners simultaneously defect or cooperate
- Players simultaneously choose rock, paper, or scissors
- Etc ...

**No** knowledge of the other players' chosen actions

What if we allow **sequential** action selection ...?

# LEADER-FOLLOWER GAMES



Heinrich von Stackelberg

Two players:

- The **leader** commits to acting in a specific way
- The **follower** observes the leader's mixed strategy

*NE, iterated strict dominance*

What is the Nash equilibrium ??????????

- Social welfare: 2
- Utility to row player: 1

Row player = leader; what to do ??????????

- Social welfare: 3
- Utility to row player: 2

| Commit to "Bottom" |      |
|--------------------|------|
| 0, 0               | 2, 1 |

# ASIDE: FIRST-MOVER ADVANTAGE (FMA)

From the econ side of things ...

- Leader is sometimes called the **Market Leader**
- Some advantage allows a firm to move first:
  - Technological breakthrough via R&D
  - Buying up all assets at low price before market adjusts

**By committing to a strategy (some amount of production), can effectively force other players' hands.**

**Things we won't model:**

- Significant cost of R&D, uncertainty over market demand, initial marketing costs, etc.

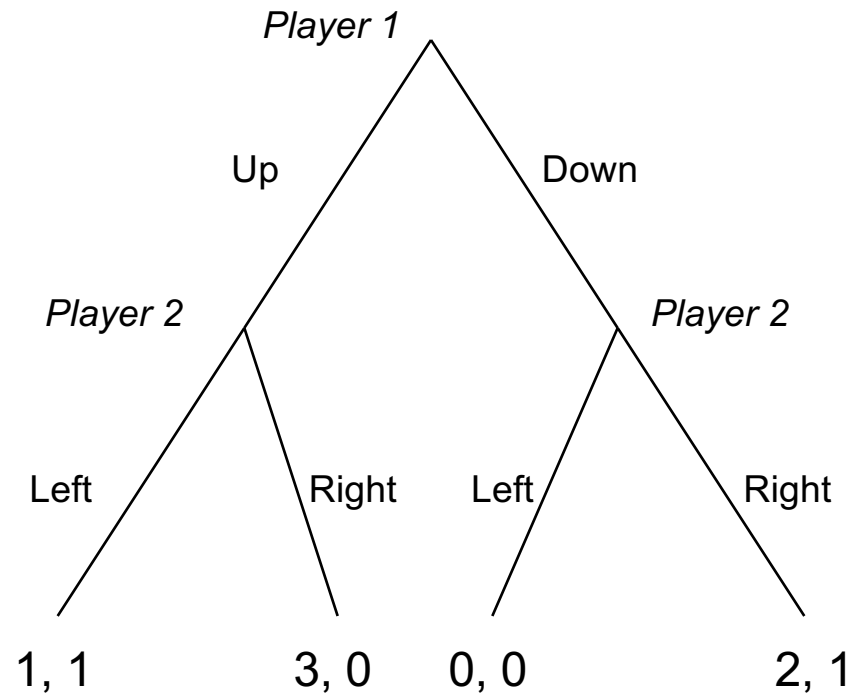
**These can lead to **Second-Mover Advantage****

- **Atari vs Nintendo, MySpace (or earlier) vs Facebook**

# COMMITMENT AS AN EXTENSIVE-FORM GAME

For the case of committing to a **pure** strategy:

|      |      |
|------|------|
| 1, 1 | 3, 0 |
| 0, 0 | 2, 1 |



# COMMITMENT TO MIXED STRATEGIES

|     | 0    | 1    |
|-----|------|------|
| .49 | 1, 1 | 3, 0 |
| .51 | 0, 0 | 2, 1 |

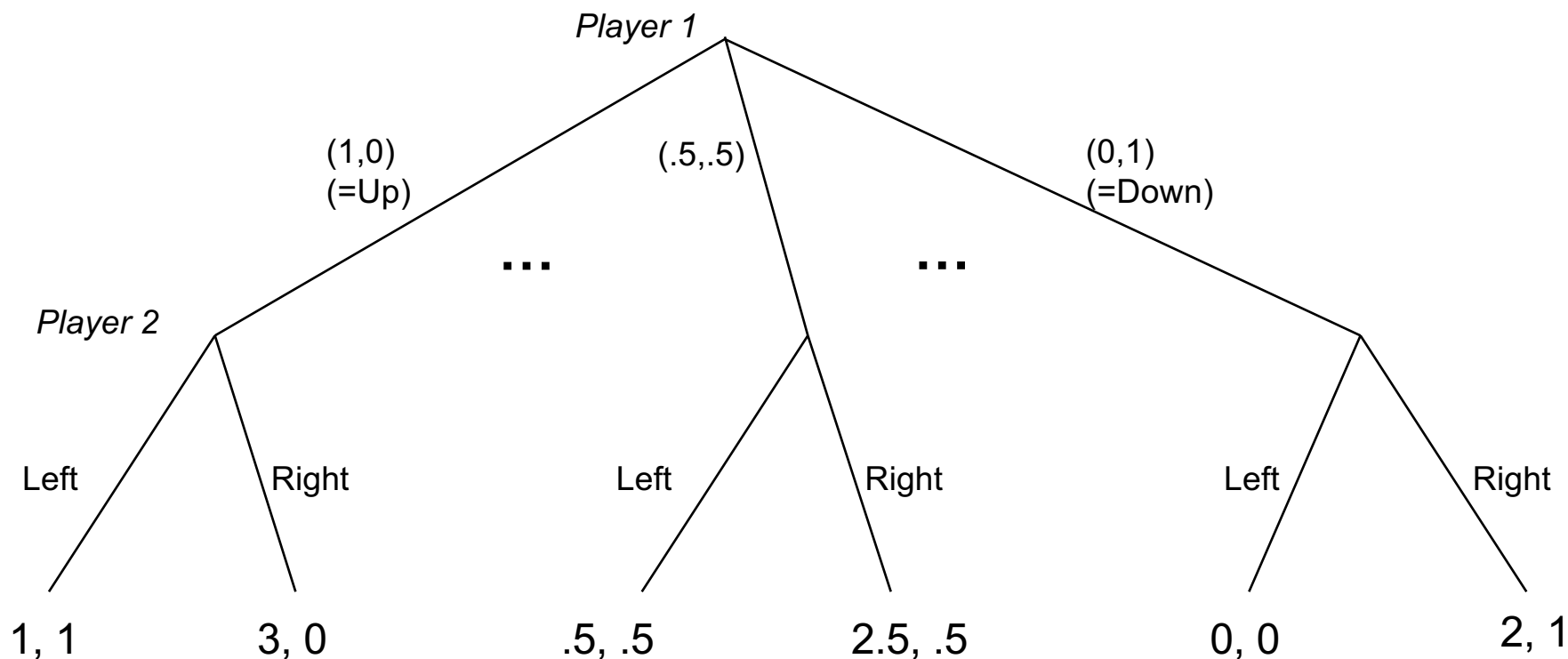
What should Column do ????????

Sometimes also called a **Stackelberg (mixed) strategy**



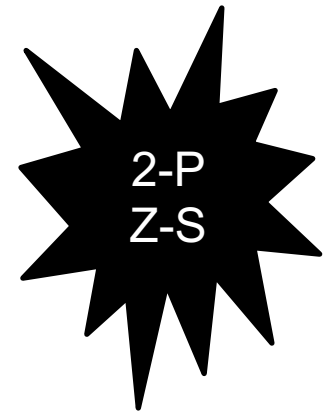
# COMMITMENT AS AN EXTENSIVE-FORM GAME...

For the case of committing to a mixed strategy:



- **Economist:** Just an extensive-form game ...
- **Computer scientist:** **Infinite-size game!** Representation matters

# WHAT SHOULD THE LEADER COMMIT TO?



Special case: 2-player zero-sum normal-form games

Recall: Row player plays Minimax strategy

- Minimizes the maximum expected utility to the Col

Doesn't matter who commits to what, when

Minimax strategies = Nash Equilibrium  
= Stackelberg Equilibrium  
(not the case for general games)

Polynomial time computation via LP – Lecture #4

# WHAT SHOULD THE LEADER COMMIT TO?



Separate LP for every column  $c^*$ :

*maximize*  $\sum_r p_r u_R(r, c^*)$  **Row utility**

*s.t.*

*for all*  $c$ ,  $\sum_r p_r u_C(r, c^*) \geq \sum_r p_r u_C(r, c)$  **Column optimality**

$\sum_r p_r = 1$

*for all*  $r$ ,  $p_r \geq 0$

**Distributional  
constraints**

**Choose strategy from LP with highest objective**

# RUNNING EXAMPLE

|   |      |      |
|---|------|------|
| x | 1, 1 | 3, 0 |
| y | 0, 0 | 2, 1 |

*maximize*  $1x + 0y$

*s.t.*

$$1x + 0y \geq 0x + 1y$$

$$x + y = 1$$

$$x \geq 0$$

$$y \geq 0$$

*maximize*  $3x + 2y$

*s.t.*

$$0x + 1y \geq 1x + 0y$$

$$x + y = 1$$

$$x \geq 0$$

$$y \geq 0$$

# IS COMMITMENT ALWAYS GOOD FOR THE LEADER?

Yes, if we allow commitment to mixed strategies

- Always weakly better to commit [von Stengel & Zamir, 2004]

What about only pure strategies?

Expected utility to Row  
by playing mixed Nash:

????????????

$$E_R[ \langle 1/3, 1/3, 1/3 \rangle ] = 0$$

Expected utility to Row by  
any pure commitment:

????????????

$$E_R[ \langle 1, 0, 0 \rangle ] = -1$$

$$E_R[ \langle 0, 1, 0 \rangle ] = -1$$

$$E_R[ \langle 0, 0, 1 \rangle ] = -1$$

|          | Rock   | Paper | Scissors |
|----------|--------|-------|----------|
| Rock     |        |       |          |
| Paper    | +1, -1 | 0, 0  | -1, +1   |
| Scissors |        |       |          |

# WHAT SHOULD THE LEADER COMMIT TO?



Bayesian games: player  $i$  draws type  $\theta_i$  from  $\Theta$

Special case: **follower has only one type**, leader has type  $\theta$

Like before, solve a separate LP for every column  $c^*$ :

$$\text{maximize } \sum_{\theta} \pi(\theta) \sum_r p_{r,\theta} u_{R,\theta}(r, c^*)$$

*s.t.*

$$\text{for all } c, \sum_{\theta} \pi(\theta) \sum_r p_{r,\theta} u_C(r, c^*) \geq \sum_{\theta} \pi(\theta) \sum_r p_{r,\theta} u_C(r, c)$$

$$\text{for all } \theta, \sum_r p_{r,\theta} = 1$$

$$\text{for all } r, \theta, p_{r,\theta} \geq 0$$

Choose strategy from LP with highest objective

# WHAT SHOULD THE LEADER COMMIT TO?



So, we showed **polynomial-time** methods for:

- 2-Player, zero-sum
- 2-Player, general-sum
- 2-Player, general-sum, Bayesian with 1-type follower

In general, **NP-hard** to compute:

- 2-Player, general-sum, Bayesian with 1-type leader
  - Arguably more interesting (“I know my own type”)
- 2-Player, general-sum, Bayesian general
- **N-Player, for  $N > 2$ :**
  - 1<sup>st</sup> player commits,  $N-1$ -Player leader-follower game, 2<sup>nd</sup> player commits, recurse until 2-Player leader-follower

# STACKELBERG SECURITY GAMES

**Leader-follower → Defender-attacker**

- Defender is interested in protecting a set of targets
- Attacker wants to attack the targets

**The defender is endowed with a set of resources**

- Resources protect the targets and prevent attacks

**Utilities:**

- Defender receives positive utility for preventing attacks, negative utility for “successful” attacks
- Attacker: positive utility for successful attacks, negative otherwise
- Not necessarily zero-sum

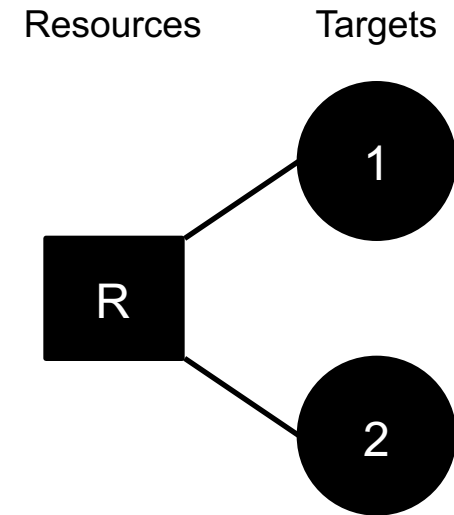


# SECURITY GAMES: A FORMAL MODEL

Defined by a 3-tuple  $(N, U, M)$ :

- **N**: set of  $n$  targets
- **U**: utilities associated with defender and attacker
- **M**: all subsets of targets that can be simultaneously defended by deployments of resources
  - A schedule  $S \subseteq 2^N$  is the set of target defended by a single resource  $r$
  - Assignment function  $A : R \rightarrow 2^S$  is the set of all schedules a specific resource can support
- Then we have  $m$  pure strategies, assigning resources such that the union of their target coverage is in  $M$
- Utility  $u_{c,d}(i)$  and  $u_{u,d}(i)$  for the defender when target  $i$  is attacked and is covered or defended, respectively

# SIMPLE EXAMPLE



| Targets | Defender     |              | Attacker Type $\theta_1$ |              | Attacker Type $\theta_2$ |              |
|---------|--------------|--------------|--------------------------|--------------|--------------------------|--------------|
| $i$     | $u_{c,d}(i)$ | $u_{u,d}(i)$ | $u_{c,a}(i)$             | $u_{u,a}(i)$ | $u_{c,a}(i)$             | $u_{u,a}(i)$ |
| 1       | 0            | -1           | 0                        | +1           | 0                        | +1           |
| 2       | 0            | -2           | 0                        | +5           | 0                        | +1           |

# REAL-WORLD SECURITY GAMES



Lots of deployed applications!

- Checkpoints at airports
- Patrol routes in harbors
- Scheduling Federal Air Marshalls
- Patrol routes for anti-poachers



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Typically solve for **strong** Stackelberg Equilibria:

- Tie break in favor of the defender; always exists
- Can often “nudge” the adversary in practice

Two big practical problems: **computation** and uncertainty

# **NEXT UP: NEAL GUPTA**

***KIEKINTVELD ET AL. COMPUTING OPTIMAL RANDOMIZED  
RESOURCE ALLOCATIONS FOR MASSIVE SECURITY GAMES.  
AAMAS-09.***