TRUSTS

Scheduling Randomized Patrols for Fare Inspection in Transit Systems

Abstract

- ‘Proof-of-payment’ transit systems:
  - Passengers: required to purchase tickets; but cannot be physically forced to do so
  - Patrol units: inspect passengers; impose fine for fare evading
- TRUSTS: application for randomized patrol strategies
  - Leader-follower Stackelberg game
  - Objective: Deter fare evasion → maximize revenue
- Major differences from previously studied Stackelberg settings
  - Leader strategies: massive temporal and spatial constraints
  - Large number of potential followers
  - Patrol strategies should be easier to execute

Introduction

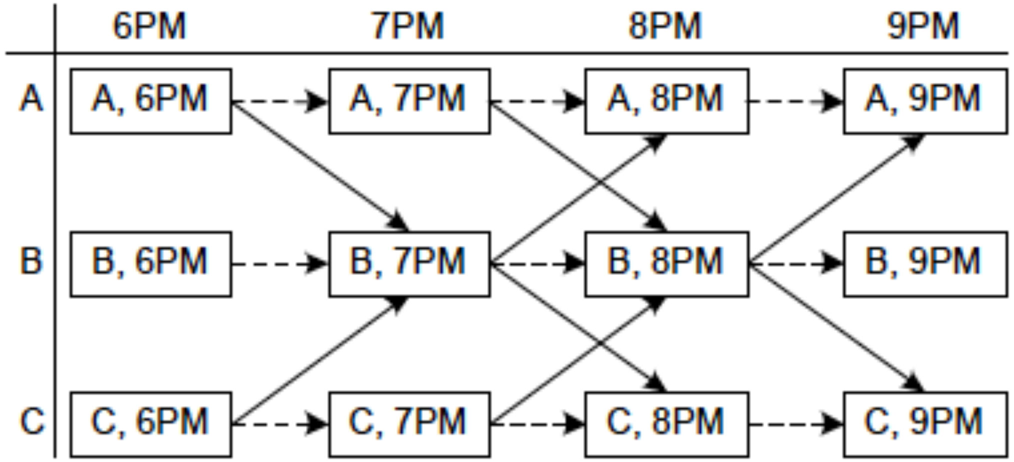
- LA Metro: ~300,000 riders daily
  - No barriers to entry → can result in significant revenue loss
  - Los Angeles Sheriffs Department (LASD) uniformed patrols on board trains and stations: relies on humans for scheduling the patrols – predictable schedules
- TRUSTS (**T**actical **R**andomization for **U**rban **S**ecurity in **T**ransit **S**ystems) application
  - Stackelberg game with 1 leader (LASD) and many followers (metro riders)
  - Leader: precommits to a mixed patrol strategy
  - Followers: observe this strategy and decide whether to buy the ticket
- Optimization objective: leader maximizes total revenue
  - Exponentially many pure strategies
- TRUSTS uses *transition graph* for spatial and temporal structure, and uses LP to optimize the flow

Problem Setting

- Generate randomized schedules for patrols for 4 separate LA Metro lines
  - Pure leader strategy: *patrol* – sequence of patrol actions with constant bounded duration
  - Pure follower strategies: buying ticket, and not buying ticket
- Train system: modeled using directed *transition graph* $G = \langle V, E \rangle$, acc. to metro timetable
  - Vertex $v = \langle s, t \rangle$: a pair of station *s* and time point *t*
  - Edge from $\langle s, t \rangle$ to $\langle s', t' \rangle$ if:
    - s and s' are consecutive stops for some train in the train schedule, or
    - $s = s'$, $t < t'$, and there is no vertex $\langle s, t'' \rangle$ with $t < t'' < t'$
- Patrols: $\boldsymbol{\gamma}$ deployable patrol units, each with patrol duration of at most $\boldsymbol{\kappa}$ hours
  - On-train inspections, in-station inspections
  - Each edge *e* has: l_e – patrol action duration; f_e – effectiveness value

Problem Setting

- Riders: assumed to be daily commuters with fixed routine
  - Ticket price: **ρ** (nominal fee); fine for fare evasion: **τ** (much larger)
  - Rider's type: defined by the path (source, destination, departure time triple)
  - Stay edge is only at the end (last edge): exiting from the destination station (i.e. rider paths are diagonal except for the last edge)
  - **Λ**: set of rider types



- Given a pure strategy of **γ** patrol units, inspection probability for rider of type $\boldsymbol{\lambda} \in \boldsymbol{\Lambda}$:
  $\min\{1, \sum_{i=1}^{\boldsymbol{\gamma}} \sum_{e \in P_i \cap \boldsymbol{\lambda}} fe\}$
- Objective:
  Leader's utility = total expected revenue = utilities from bilateral interactions with individual followers (zero-sum game)
  - Equivalent to Bayesian Stackelberg game: 1 leader of 1 type, 1 follower of multiple types

Linear Program Formulation

- Find maximum revenue (mixed) patrol strategy
  - Leader's pure strategies: exponentially large
  - Represent mixed strategies by marginal coverage on edges x_e of the transition graph (expected no. of inspections on these edges)

- Basic Formulation:
  - For the transition graph $G = \langle V, E \rangle$:
    $V^+ \subset V$: set of possible starting vertices; $V^- \subset V$: set of possible ending vertices
  - Add source v^+ and sink v^-; edges from v^+ to V^+, and from V^- to v^-
  - u_λ: expected value paid by rider of type λ

Basic Formulation

$$\max_{\mathbf{x},\mathbf{u}} \sum_{\lambda\in\Lambda} p_\lambda u_\lambda$$

Expected total revenue from riders of type **λ**

$$\text{s.t. } u_\lambda \leq \min\{\rho, \tau \sum_{e\in\lambda} x_e f_e\}, \text{ for all } \lambda \in \Lambda$$

$$\sum_{v\in V^+} x_{(v^+,v)} = \sum_{v\in V^-} x_{(v,v^-)} \leq \gamma$$

$$\sum_{(v',v)\in E} x_{(v',v)} = \sum_{(v,v^\dagger)\in E} x_{(v,v^\dagger)}, \text{ for all } v \in V$$

Conservation of flow

$$\sum_{e\in E} l_e \cdot x_e \leq \gamma \cdot \kappa, 0 \leq x_e \leq \alpha, \forall e \in E$$

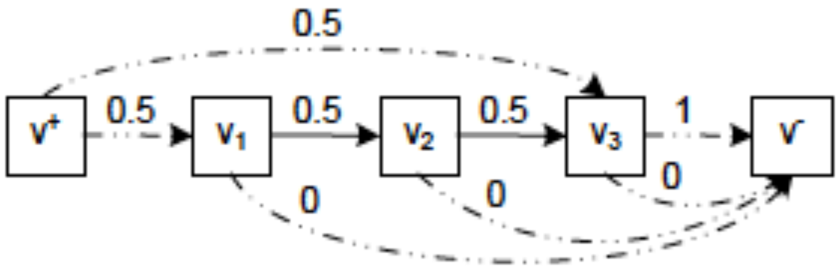
α∈[1, **γ**] : used as an upper-bound on x_e
limits the number of patrol units on an edge

Results in an **overestimate** of the actual inspection probability, and thus leader's utility

Issues with Basic Formulation

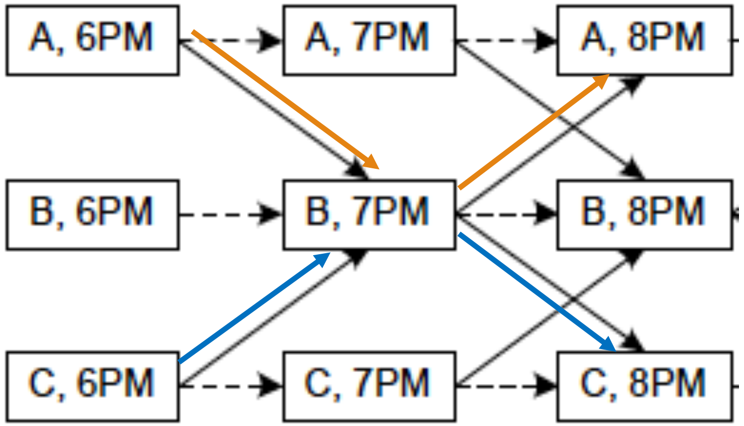
2 fundamental issues:

- Failure to satisfy patrol length limit constraint

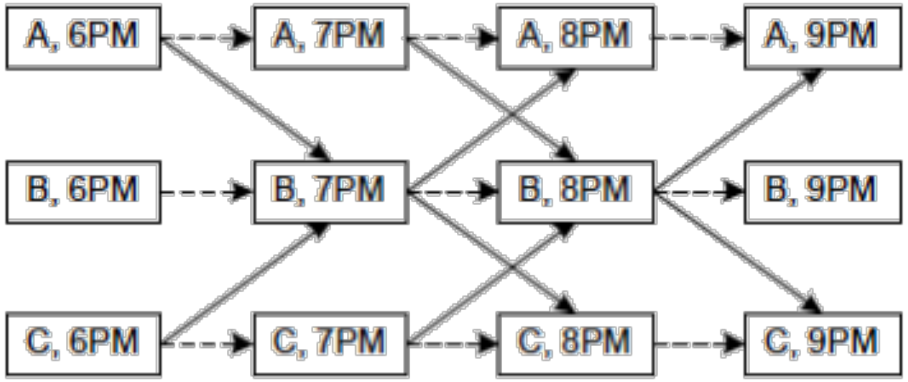


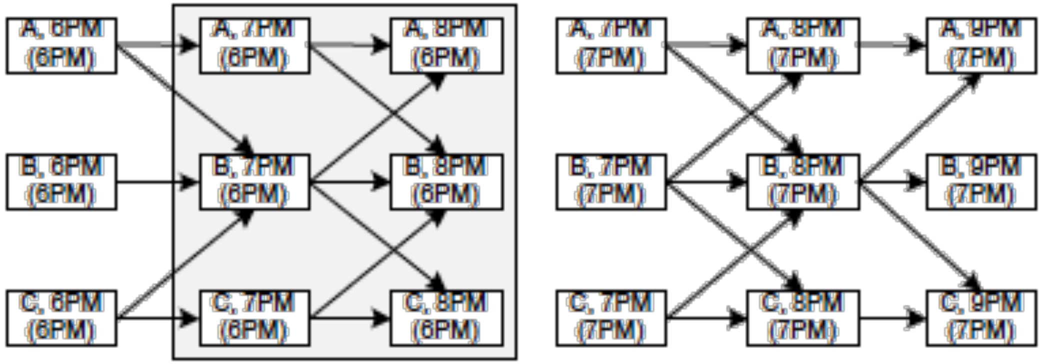
Assume γ = 1 and κ = 1

- Frequent/impractical switching between trains or in-station/on-train patrol



Extended Formulation



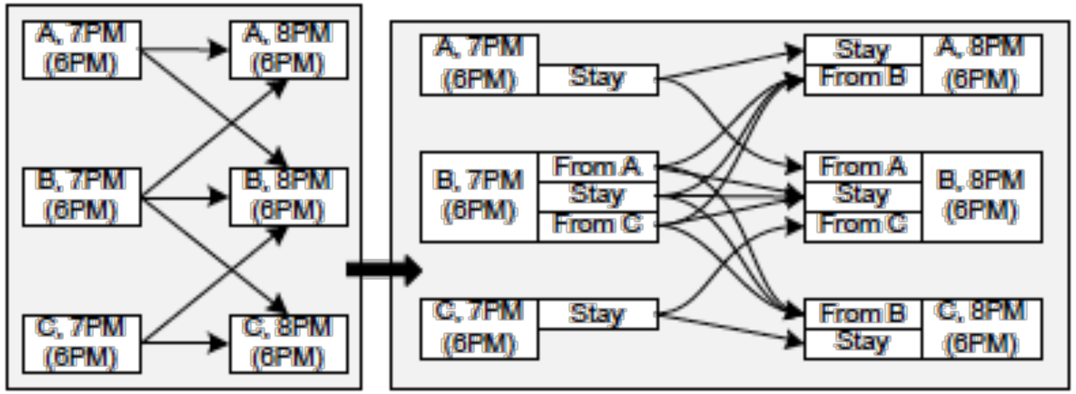


(a) $\kappa = 2$

History-Duplicate Transition (HDT) graph: $\mathcal{G} = \langle \mathcal{V}, \mathcal{E} \rangle$

- Multiple restricted copies of G
- For starting point t*, keep only the vertices $v = \langle s, t \rangle \in V$, where $t^* \leq t \leq t^* + \boldsymbol{\kappa}$
- Linear expansion of G: at most $\lceil \boldsymbol{\kappa}/\boldsymbol{\delta} \rceil$

  $\boldsymbol{\delta}$: difference between starting time points



(b)

Extended Formulation

$$\max_{\mathbf{x},\mathbf{y},\mathbf{u}} \sum_{\lambda\in\Lambda} p_\lambda u_\lambda - \beta \sum_{e\in\mathcal{E}} c_e y_e$$

Used to penalize the switching edges

c_e = 1 for switching edge, 0 otherwise

$$\text{s.t. } u_\lambda \leq \min\{\rho,\ \tau \sum_{e\in\lambda} x_e f_e\}, \text{ for all } \lambda \in \Lambda$$

$$\sum_{v\in\mathcal{V}^+} y_{(v^+,v)} = \sum_{v\in\mathcal{V}^-} y_{(v,v^-)} \leq \gamma$$

y_e: represent marginal coverage in HDT

$$\sum_{(v',v)\in\mathcal{E}} y_{(v',v)} = \sum_{(v,v^\dagger)\in\mathcal{E}} y_{(v,v^\dagger)}, \text{ for all } v \in \mathcal{V}$$

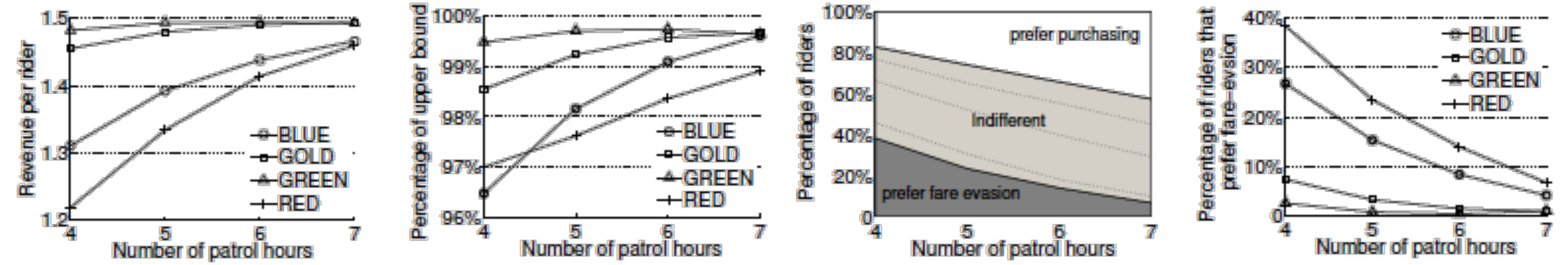
$$x_e = \sum_{e'\in\Gamma(e)} y_{e'}, \forall e \in E, 0 \leq x_e \leq \alpha, \forall e \in E$$

Convert back to get marginal coverage for original graph

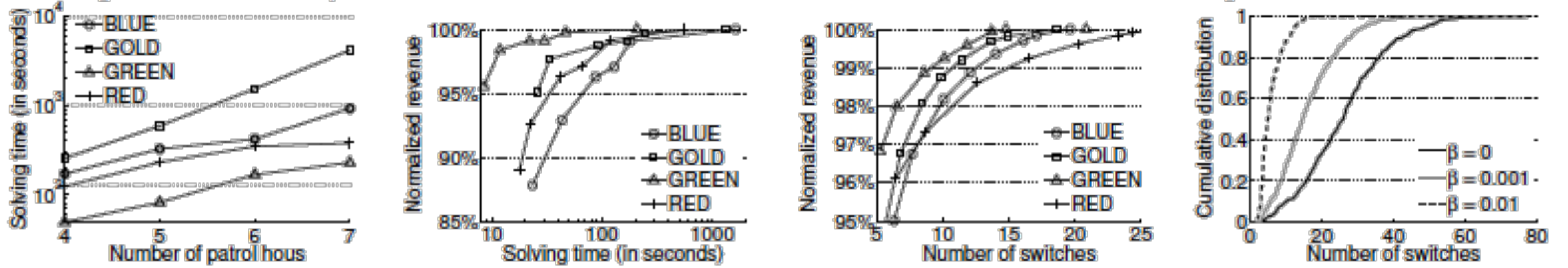
β=0 provides a tighter upper bound on the optimal revenue than that in Basic Formulation

Experimental Results

- 4 data sets, 1 each for LA Metro Rail line; **γ** = 1, **δ** = 1 hour, **κ** = 4 – 7 hours



(a) Per passenger revenue of the computed mixed strategy. (b) Percentage vs. upper bound. (c) Evasion tendency distribution of Red line. (d) Percentage of riders that prefer fare evasion



(e) Runtime of solving the LP by CPLEX. (f) Tradeoffs between optimality and runtime. (g) Tradeoffs between optimality and patrol preference. (h) Cumulative probability distribution of the number of switches for the Red line.

Thanks