

Hypothesis Testing II

?10/10/1977?



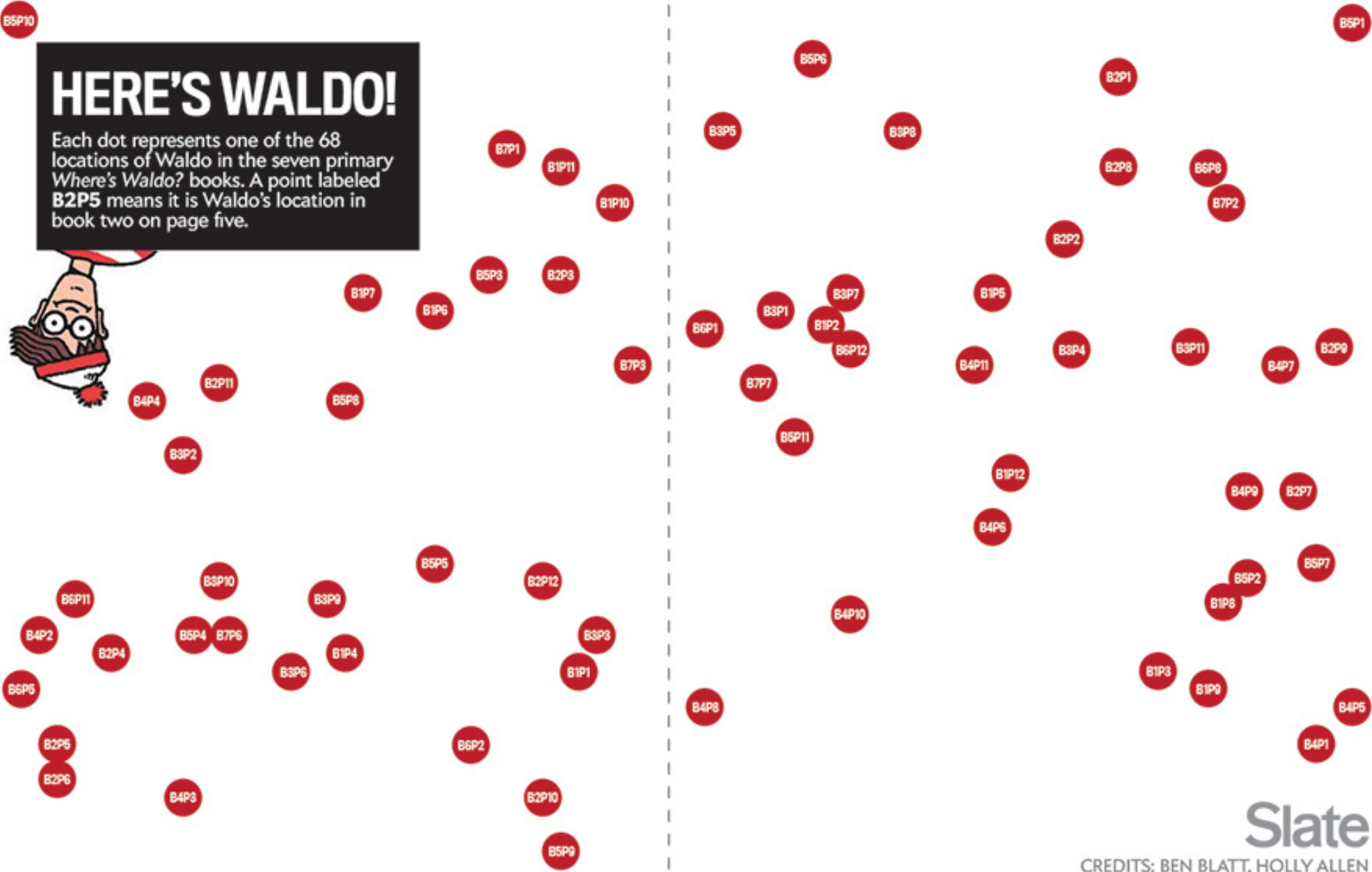


Waldo Stats. Is this a normal distribution?

B5P10

HERE'S WALDO!

Each dot represents one of the 68 locations of Waldo in the seven primary *Where's Waldo?* books. A point labeled **B2P5** means it is Waldo's location in book two on page five.

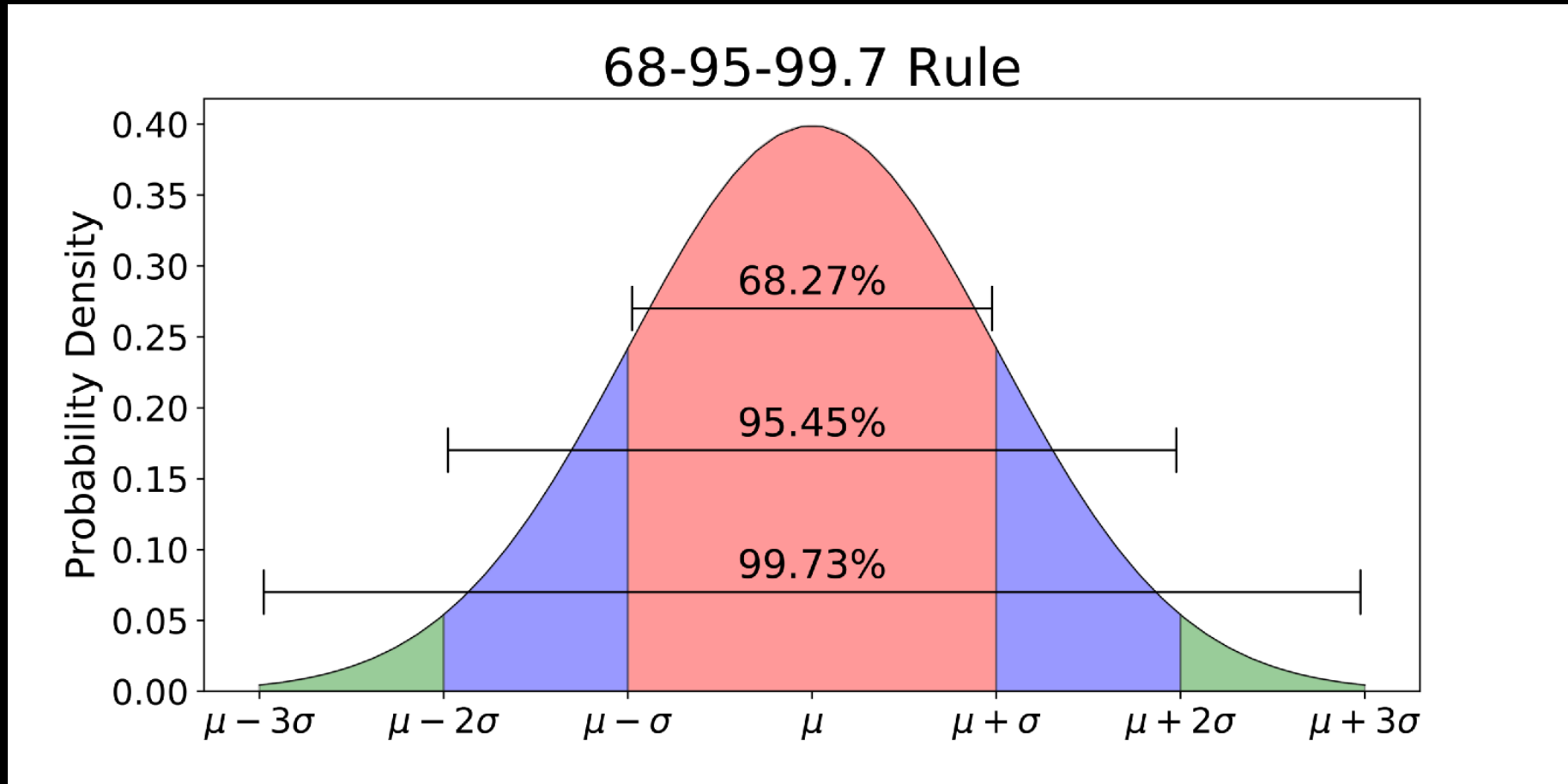


B5P1

Slate

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Standard deviations and probability of the population mean



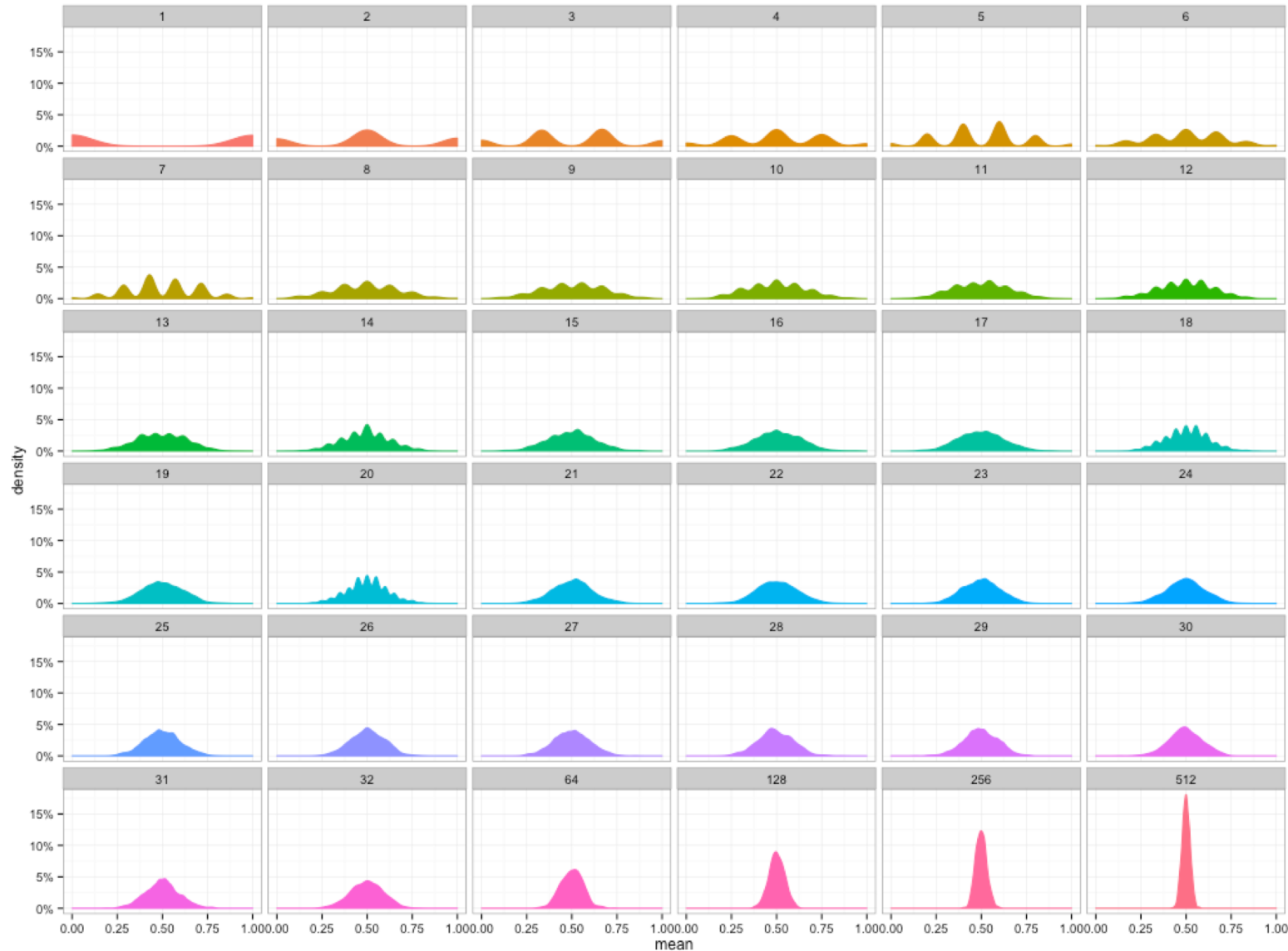
The process of testing hypothesis

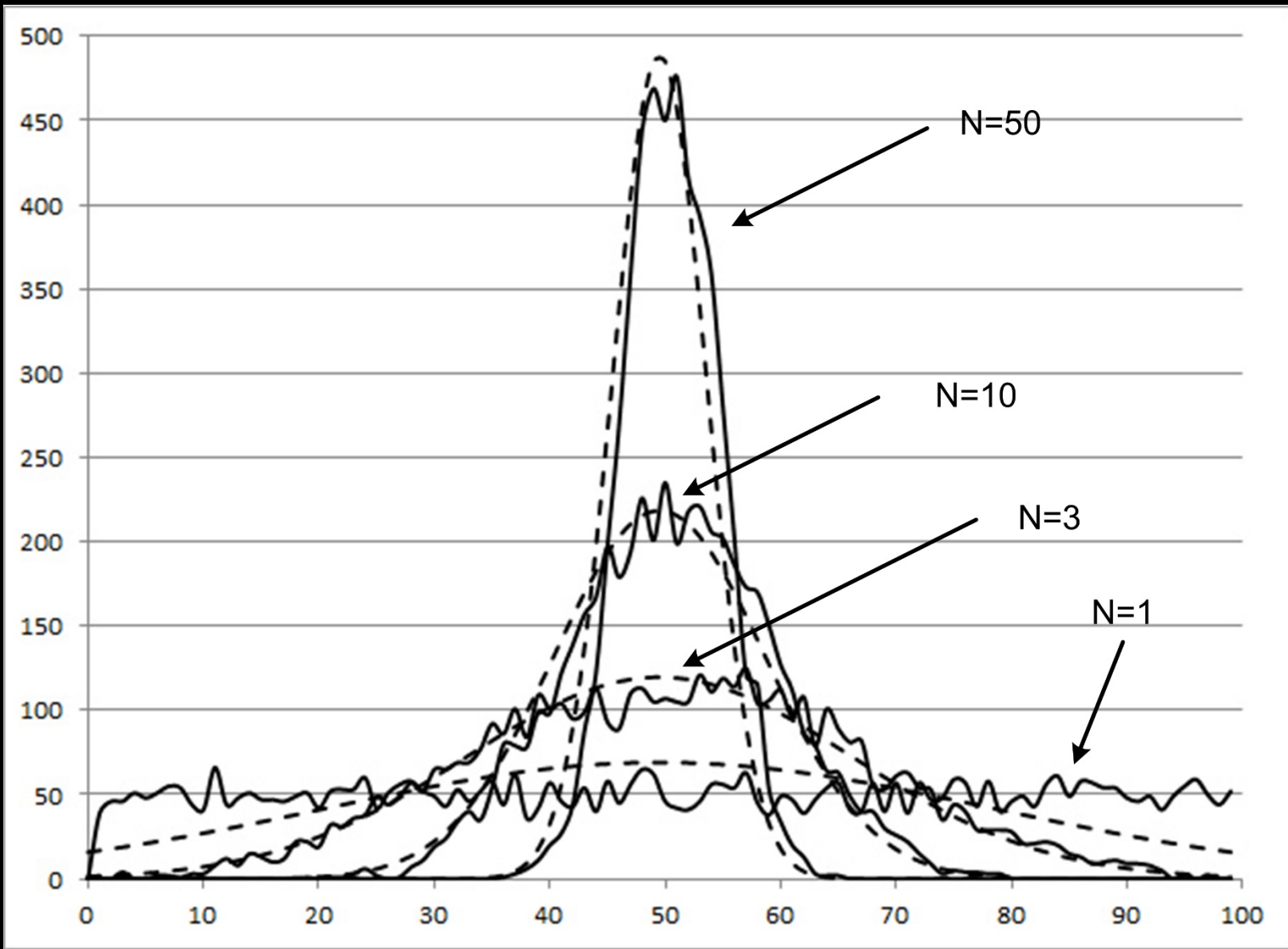
1. Examine null hypothesis
2. If suspicious, try to disprove it (where's the confidence numbers)
3. Decide how to sample randomly
4. Figure out how many N to sample?
5. Calculate mean
6. Calculate standard deviation
7. Calculate Z score (how many deviations away are you)
8. Reject or accept based on probability usually $\geq 95\%$, definitely if over 99.7%

How many N to sample?

More is better

1. Cost + time a factor
2. As N increases the standard deviation decreases
3. Confidence interval decreases, +- error





Z-Values

A z-value gives you p-value (probability of getting this value)

This is a cumulative from the mean so multiply your $Z*2$ to get a total probability. E.g., $z=2 \rightarrow p = .47*2 = 95\%$

z	+0.00	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
0.0	0.00000	0.00399	0.00798	0.01197	0.01595	0.01994	0.02392	0.02790	0.03188	0.03586
0.1	0.03983	0.04380	0.04776	0.05172	0.05567	0.05962	0.06356	0.06749	0.07142	0.07535
0.2	0.07926	0.08317	0.08706	0.09095	0.09483	0.09871	0.10257	0.10642	0.11026	0.11409
0.3	0.11791	0.12172	0.12552	0.12930	0.13307	0.13683	0.14058	0.14431	0.14803	0.15173
0.4	0.15542	0.15910	0.16276	0.16640	0.17003	0.17364	0.17724	0.18082	0.18439	0.18793
0.5	0.19146	0.19497	0.19847	0.20194	0.20540	0.20884	0.21226	0.21566	0.21904	0.22240
0.6	0.22575	0.22907	0.23237	0.23565	0.23891	0.24215	0.24537	0.24857	0.25175	0.25490
0.7	0.25804	0.26115	0.26424	0.26730	0.27035	0.27337	0.27637	0.27935	0.28230	0.28524
0.8	0.28814	0.29103	0.29389	0.29673	0.29955	0.30234	0.30511	0.30785	0.31057	0.31327
0.9	0.31594	0.31859	0.32121	0.32381	0.32639	0.32894	0.33147	0.33398	0.33646	0.33891
1.0	0.34134	0.34375	0.34614	0.34849	0.35083	0.35314	0.35543	0.35769	0.35993	0.36214
1.1	0.36433	0.36650	0.36864	0.37076	0.37286	0.37493	0.37698	0.37900	0.38100	0.38298
1.2	0.38493	0.38686	0.38877	0.39065	0.39251	0.39435	0.39617	0.39796	0.39973	0.40147
1.3	0.40320	0.40490	0.40658	0.40824	0.40988	0.41149	0.41308	0.41466	0.41621	0.41774
1.4	0.41924	0.42073	0.42220	0.42364	0.42507	0.42647	0.42785	0.42922	0.43056	0.43189
1.5	0.43319	0.43448	0.43574	0.43699	0.43822	0.43943	0.44062	0.44179	0.44295	0.44408
1.6	0.44520	0.44630	0.44738	0.44845	0.44950	0.45053	0.45154	0.45254	0.45352	0.45449
1.7	0.45543	0.45637	0.45728	0.45818	0.45907	0.45994	0.46080	0.46164	0.46246	0.46327
1.8	0.46407	0.46485	0.46562	0.46638	0.46712	0.46784	0.46856	0.46926	0.46995	0.47062
1.9	0.47128	0.47193	0.47257	0.47320	0.47381	0.47441	0.47500	0.47558	0.47615	0.47670
2.0	0.47725	0.47778	0.47831	0.47882	0.47932	0.47982	0.48030	0.48077	0.48124	0.48169
2.1	0.48214	0.48257	0.48300	0.48341	0.48382	0.48422	0.48461	0.48500	0.48537	0.48574
2.2	0.48610	0.48645	0.48679	0.48713	0.48745	0.48778	0.48809	0.48840	0.48870	0.48899
2.3	0.48928	0.48956	0.48983	0.49010	0.49036	0.49061	0.49086	0.49111	0.49134	0.49158
2.4	0.49180	0.49202	0.49224	0.49245	0.49266	0.49286	0.49305	0.49324	0.49343	0.49361
2.5	0.49379	0.49396	0.49413	0.49430	0.49446	0.49461	0.49477	0.49492	0.49506	0.49520
2.6	0.49534	0.49547	0.49560	0.49573	0.49585	0.49598	0.49609	0.49621	0.49632	0.49643
2.7	0.49653	0.49664	0.49674	0.49683	0.49693	0.49702	0.49711	0.49720	0.49728	0.49736
2.8	0.49744	0.49752	0.49760	0.49767	0.49774	0.49781	0.49788	0.49795	0.49801	0.49807
2.9	0.49813	0.49819	0.49825	0.49831	0.49836	0.49841	0.49846	0.49851	0.49856	0.49861
3.0	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49896	0.49900
3.1	0.49903	0.49906	0.49910	0.49913	0.49916	0.49918	0.49921	0.49924	0.49926	0.49929
3.2	0.49931	0.49934	0.49936	0.49938	0.49940	0.49942	0.49944	0.49946	0.49948	0.49950
3.3	0.49952	0.49953	0.49955	0.49957	0.49958	0.49960	0.49961	0.49962	0.49964	0.49965
3.4	0.49966	0.49968	0.49969	0.49970	0.49971	0.49972	0.49973	0.49974	0.49975	0.49976
3.5	0.49977	0.49978	0.49978	0.49979	0.49980	0.49981	0.49981	0.49982	0.49983	0.49983
3.6	0.49984	0.49985	0.49985	0.49986	0.49986	0.49987	0.49987	0.49988	0.49988	0.49989
3.7	0.49989	0.49990	0.49990	0.49990	0.49991	0.49991	0.49992	0.49992	0.49992	0.49992
3.8	0.49993	0.49993	0.49993	0.49994	0.49994	0.49994	0.49994	0.49995	0.49995	0.49995
3.9	0.49995	0.49995	0.49996	0.49996	0.49996	0.49996	0.49996	0.49996	0.49997	0.49997
4.0	0.49997	0.49997	0.49997	0.49997	0.49997	0.49997	0.49998	0.49998	0.49998	0.49998

T-Values
 If $N < 30$ then the standard deviations become larger since n is smaller. So it now takes 2.262 deviations to get 95% of the data.

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$	
	one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001	
df												
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62	
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599	
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924	
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610	
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869	
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959	
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408	
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041	
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781	
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587	
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437	
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318	
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221	
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140	
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073	
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015	
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965	
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922	
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883	
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850	
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819	
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792	
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768	
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745	
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725	
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707	
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690	
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674	
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659	
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646	
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551	
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460	
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416	
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390	
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300	
Z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291	
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%	
	Confidence Level											

Any statistic has an error +/- (we're speculating)

We call this the margin of error

For the mean we have:

Mean $\pm Z^*(\text{Standard Deviation}/\text{sqrt}(N))$

$$\bar{X} \pm Z \frac{S}{\sqrt{n}}$$

E.g., for the Query analysis we did we have

Mean = 4, Z=-3.40, S=1.76, so we have

4 \pm 1.09 @99.97%. So we can say we are 999.7% confident that population mean is 4 \pm 1.09 or we are 999.7% confident that the population mean is within the confidence interval 2.91 and 5.09

Margin of Error continued

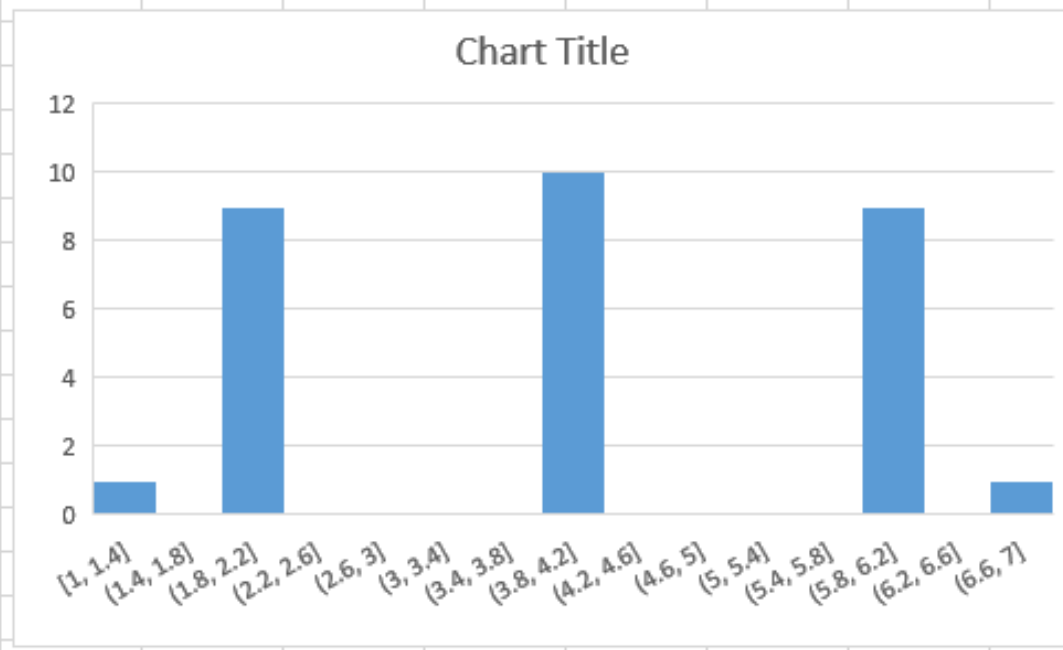
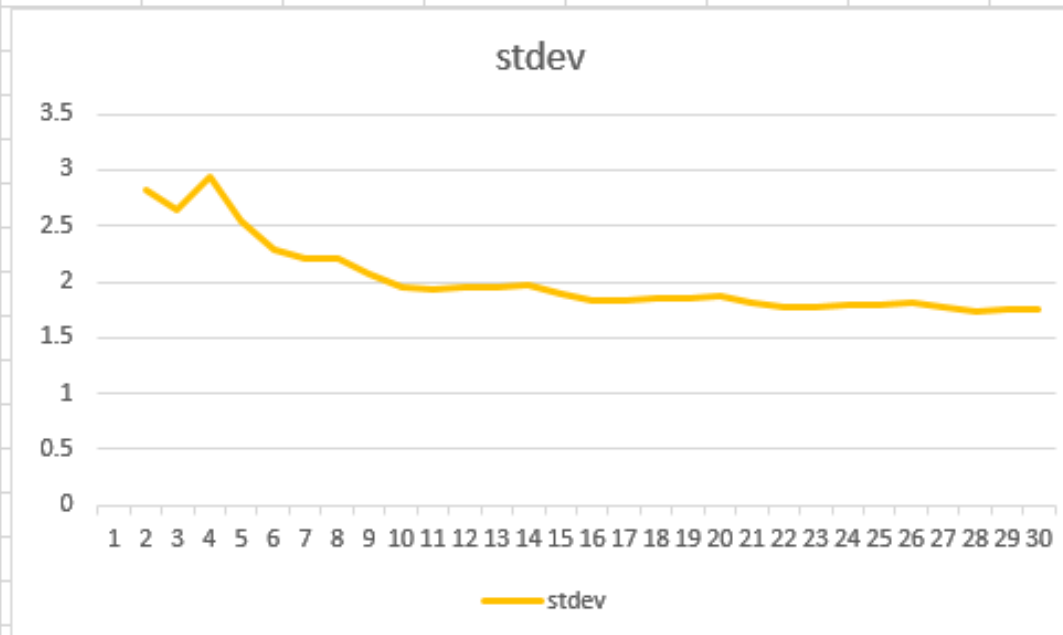
$$\bar{X} \pm Z \frac{S}{\sqrt{n}}$$

We could have chosen a 95% margin of error also and by doing so our confidence interval would have decreased. E.g.,

Mean = 4, Z=2, S=1.76, so we have

4 +/- .64 @95%. So we can say we are 95% confident that population mean is 4 +/- .64 or we are 95% confident that the population mean is within the confidence interval 3.36 and 4.64. This is how you should present your findings. Statistic, N, Standard Deviation and confidence numbers.

running mean	measure nearest minute	n	stdev	z-score	confidence	
10	2	2	1		0	
	4	6	2	2.828427	-2.12132	-4.24264
	3.5	1	3	2.645751	-2.456769	-3.75278
	4	7	4	2.94392	-2.038099	-3
	5.5	4	5	2.54951	-1.765045	-2.01246
	4	4	6	2.280351	-2.631174	-2.44949
	5	6	7	2.21467	-2.257673	-1.88982
	4	2	8	2.203893	-2.722456	-2.12132
	3	4	9	2.061553	-3.395499	-2.33333
	4	4	10	1.943651	-3.086975	-1.89737
	3	2	11	1.940009	-3.60823	-2.11058
	4	6	12	1.954017	-3.070598	-1.73205
	6	6	13	1.951331	-2.049883	-1.1094
	4	2	14	1.961161	-3.059412	-1.60357
	3	4	15	1.889822	-3.704052	-1.80739
	4	4	16	1.825742	-3.286335	-1.5
	3	2	17	1.833111	-3.818646	-1.69775
	4	6	18	1.847096	-3.248342	-1.41421
	6	6	19	1.852768	-2.158932	-0.91766
	4	2	20	1.863782	-3.21926	-1.34164
	3	4	21	1.81659	-3.853373	-1.52753
	4	4	22	1.772811	-3.384456	-1.2792
	3	2	23	1.781548	-3.929167	-1.4596
	4	6	24	1.793709	-3.345025	-1.22474
	6	6	25	1.800926	-2.22108	-0.8
	4	2	26	1.811077	-3.312946	-1.1767
	3	4	27	1.775907	-3.941648	-1.34715
	4	4	28	1.74271	-3.442914	-1.13389
	3	2	29	1.751143	-3.997389	-1.29987
final	4	6	30	1.761661	-3.405877	-1.09545



Type I & Type II errors

Null Hypothesis True	Did we reject	Example	Outcome	Probability we're right	Probability wrong
t	t	They say 10 minutes is the best, we say 4 but in fact they're correct (maybe we needed more N)	Type 1 error	99.970%	0.030%
t	f	They say 10 minutes is the best, we say 9 but 9 is within 95% so the null hypothesis holds.	Great, should have accepted it	94.900%	5.100%
f	f	They say 10 minutes is the best, we say 9 but 9 is within 95% so the null hypothesis holds. We are both wrong proven with higher N or another study is conducted and finds we sampled poorly	Type 2 error	94.900%	5.100%
f	t	They say 10 minutes is the best, we say 4 and we're correct (we increased N later on and verified)	Great, should have rejected it	99.970%	0.030%

Some examples

- The average time it takes to find Waldo at Comic Con by a random group of people over age 6 is 7 minutes. We surveyed 30 random people over age six and they found Waldo in an average time of 5 minutes. Who is right?
- Null hypo = mean = 7
- New hypo = mean = 5. $N = 30$, calculated sdev = 2.08, Z-Score = $-.96$
- Z-table look up gives = $.33 * 2 \rightarrow p = 66\%$. So there's a 66% chance that the population mean is in this range. Margin of error = $5 \pm .36$
66% confidence. Should we reject?

Time permitting in class examples with Waldo

