

1. Assume that each word of your machine has 64 bits. Assume that you can multiply two  $n$ -word numbers in time  $3n^2$  with a standard algorithm. Assume that you can multiply two  $n$ -word numbers in time  $8n^{\lg 3}$  with a “fancy” algorithm. For each part *briefly justify* and *show your work*.
  - (a) Approximately, how large does  $n$  have to be for the fancy algorithm to be better?
  - (b) How many bits is that?
  - (c) How many decimal digits is that?

2. Consider the following recurrence (for the time of some algorithm).

$$T(n) = 3T(n/5) + 2n + 1, \quad T(1) = 4.$$

- (a) Calculate  $T(25)$  by hand. Show your work.
  - (b) Use the tree method to solve the recurrence exactly, assuming  $n$  is a power of 5. For each subpart *briefly justify* and/or *show your work* when appropriate.
    - (i) Draw the tree. You should show at least three levels at the top and at least two levels at the bottom (as done in class).
    - (ii) What is the height of the tree? (Note that a tree with one node has height 0, a tree with a root and some children has height 1, etc.)
    - (iii) How many leaves are there?
    - (iv) What is the total work done by the leaves?
    - (v) What is the size of each subproblem at level  $i$ ? (Note that the root is at level 0, its children are at level 1, etc.)
    - (vi) How much work does each subproblem at level  $i$  (above the leaves) do?
    - (vii) What is the total work for level  $i$  (above the leaves)?
    - (viii) Write a summation for the total work not including the leaves?
    - (ix) Simplify the summation.
    - (x) What is the total work for the entire algorithm?
3. Selection Sort finds the largest element and puts it at the end of the array. Consider a version of Selection Sort that finds the two largest elements and puts both of them at the end of the array (in order).
  - (a) Write the pseudo-code for this version of Selection Sort. Make sure that it works when the size of the array is odd.
  - (b) Calculate the exact worst-case number of comparisons. Show your work.