

1. One way to find the median of a list is to sort the list and then take the middle element.
  - (a) Assume you use Bubble Sort to sort a list with 7 elements (i.e.  $n = 7$ ). Exactly how many comparisons do you use (in the worst case)?
  - (b) Assume you use Mergesort to sort a list with 7 elements (i.e.  $n = 7$ ). Exactly how many comparisons do you use (in the worst case)?
2. You can actually find the median by running a sorting algorithm and stopping early, as soon as you know the median.
  - (a) Assume you use Bubble Sort to find the median of 7 elements (i.e.  $n = 7$ ), but stop as soon as you know the median. Exactly how many comparisons do you use (in the worst case)?
  - (b) Assume you use Mergesort to find the median of 7 elements (i.e.  $n = 7$ ), but stop as soon as you know the median. Exactly how many comparisons do you use (in the worst case)?
3. The selection algorithm (to find the  $k$ th smallest value in a list), described in the class (and in the book), uses columns of size 5. Assume you implement the same selection algorithm using columns of size 7, rather than 5.
  - (a) Exactly how far from either end of the array is the median of medians guaranteed to be. Just give the high order term. (Recall that with columns of size 5 we got  $\frac{3n}{10}$ .)
  - (b) Write down the recurrence for a Selection algorithm based on columns with 7 elements each, using (the full) bubble sort to find the median of each column. (You can ignore floors and ceilings, as we did in class.) You do not have to give the algorithm, but state where each of the terms in your recurrence comes from. (For example, you might say that the  $n - 1$  term comes from partition.)
  - (c) Solve the recurrence, and give the high order term exactly.
  - (d) How does this value compare with what we got in class for columns of size 5?
4. Assume that you implement Selection Sort using the recursive algorithm on Homework 5.
  - (a) Write a recurrence for exactly how many comparisons it uses.
  - (b) Solve the recurrence any way you like. Show your work.
5. Let  $G = (V, E)$  be a directed graph.
  - (a) Assuming that  $G$  is represented by a 2-dimensional adjacency matrix  $A[1..n, 1..n]$ , give a  $\Theta(n^2)$ -time algorithm to compute the adjacency list representation of  $G$ , with  $A[i, j]$  representing an edge between  $i$  and  $j$  vertices. (Represent the addition of an element(vertex),  $v$ , to an adjacency list,  $l$ , using pseudo code,  $l \leftarrow l \cup \{v\}$ .)
  - (b) Assuming that  $G$  is represented by an adjacency list  $\text{Adj}[1..n]$ , give a  $\Theta(n^2)$ -time algorithm to compute the 2-dimensional adjacency matrix representation of  $G$ .