CMSC 425 : Fall 2018 Dave Mount

Homework 1: Geometric Computation

Due: Tue, Oct 23, 9:30am. Submission instructions at the end.

Late policy: Since solutions will be discussed in class on the due date, no late homeworks will be accepted. Turn in whatever you have done by the due date.

Problem 1. For each of the following tasks, express the answer as a function of primitive geometric operations: point-vector arithmetic, affine combinations, dot product, cross product, orientation, affine transformations, quaternions. You may also use standard math functions such as square-root and trigonometric functions. (Please *avoid high-level operations*, such as distance(p, q) and *do not given Unity functions.*)

(Sample) Given a nonzero vector $\vec{v}$ in 3-space, compute a vector $\vec{u}$ of *unit length* that points in the *opposite direction* as $\vec{v}$. (*Answer*: $-\vec{v}/\sqrt{\vec{v} \cdot \vec{v}}$.)

(i) Given two vectors $\vec{u}$ and $\vec{v}$ in 3-space, compute the *angle* between them in *degrees*.

(ii) Given two vectors $\vec{u}$ and $\vec{v}$ in 3-space (not parallel to each other), compute a third vector $\vec{w}$ that is of *unit length* and *perpendicular* to both of them.

(iii) Given a point p that lies on the (x, z)-plane (the ground), a directional unit vector $\vec{u}$ in the (x, z)-plane, and two positive scalars f and h, compute a point c that lies f units behind p (relative to the forward direction $\vec{u}$) and h units above the ground (see Fig. 1(a)). (This is useful for placing a camera that follows a game object moving in direction $\vec{u}$.)

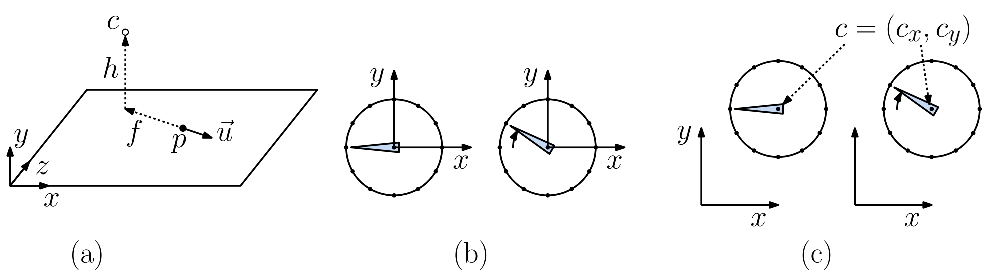

Figure 1: (a) Problem 1(iii), (b) Problem 1(iv), and (c) Problem 1(v).

(iv) Given analog clock in the 2-dimensional (x, y)-plane with its hands centered at the origin, compute an affine transformation that rotates the minute hand of the clock forward (clockwise) by 5 minutes (see Fig. 1(b)). (Assume that points are represented as a 3-element column vector in homogeneous coordinates, and express your answer as a 3×3 matrix).

(v) Same as (iv), but now the center of the clock is at the point $c = (c_x, c_y)$ (see Fig. 1(c)). (Use the same representation as in (iv). You may express your answer either as a single matrix or as the product of multiple matrices.)

Problem 2. You have been asked to develop a program that performs *mesh refinement*, splitting coarse meshes into finer ones. Your first task is to subdivide one triangle. Given a triangle in 3-space with vertices a, b, and c (see Fig. 2(a)), your job is to compute the seven vertices that result if the triangle is subdivided into 9 triangles, each of 1/3 the size of the original triangle by trisecting each of the sides of the triangle (see Fig. 2(b)).

Using *affine combinations*, explain how to compute each of the seven vertices in terms of the original vertices a, b, and c. Please use the naming convention shown in the figure for these vertices.

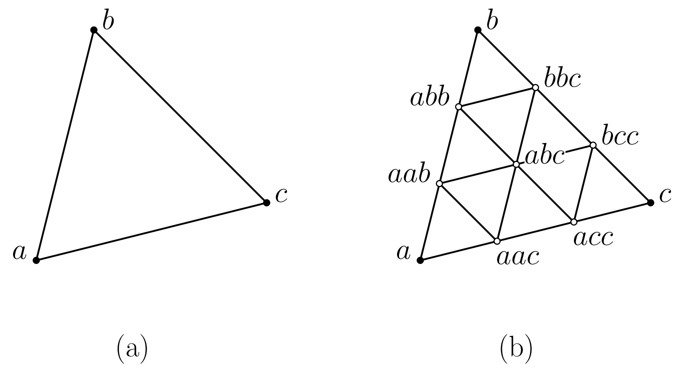

Figure 2: Problem 2: Triangle-mesh refinement.

Problem 3. Let us consider a variant of the projectile-shooting problem given in class (see Lecture 8). A projectile is launched from a location that is h meters above the ground. Let us assume Unity's convention, where the ground is the (x, z)-plane. Let (p_x, h, p_z) denote the point where the projectile is launched. Let $\vec{v}_0 = (v_{0,x}, v_{0,y}, v_{0,z})$ be the projectile's initial velocity (see Fig. 3). As in the lecture, you may assume that $g \approx 9.8\text{m/sec}^2$ denotes the acceleration due to gravity.

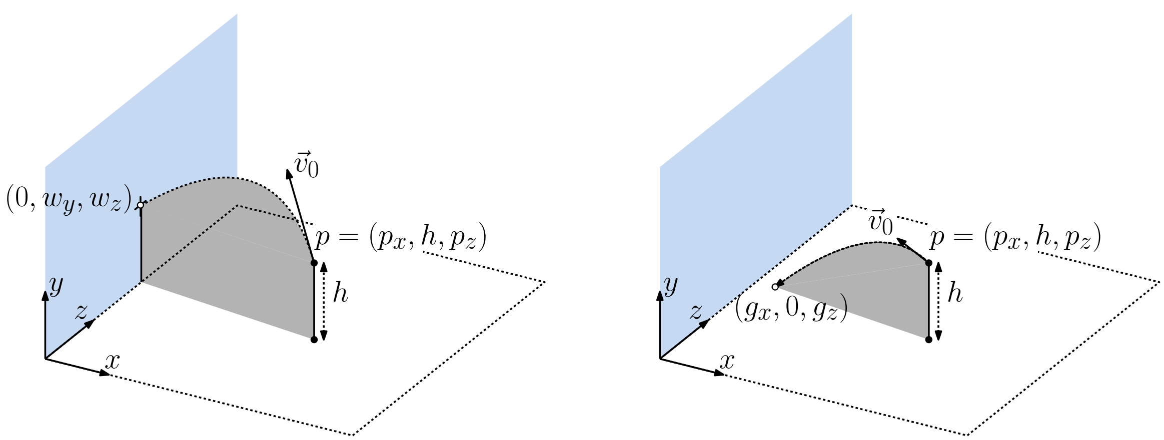

Figure 3: Problem 3: Projectile shooting revisited.

Using mathematical notation, write an expression that determines whether the projectile hits the (y, z)-coordinate plane first or hits the ground first. If it hits the (y, z) plane first, derive the coordinates (w_y, w_z) where it hits this plane. If it hits the ground first, derive the coordinates (g_x, g_z) where it hits the ground.

Challenge Problem. (Challenge problems are not included in the homework score. They count for extra-credit points, which are considered at the end of the semester after the final grade cutoffs have been computed.)

You are working on a game in an urban setting, and you are assigned the job of writing a program that simulates how pedestrians walk. One feature of natural human walking behavior is that they anticipate collisions and try to avoid them. In this problem, we will just consider the simple question of detecting whether two pedestrians might collide.

You are given two circular disks in the (x, y)-plane, both of radius 1. The first is centered at a point $p = (p_x, p_y)$ and is traveling at velocity $\vec{u} = (u_x, u_y)$. The second is centered at a point $q = (q_x, q_y)$ and is traveling at velocity $\vec{v} = (v_x, v_y)$. After t time units, the first disk is centered at the point $p + t\vec{u}$ and the second is at $q + t\vec{v}$.

Given p, q, $\vec{u}$ and $\vec{v}$ write a mathematical expression that determines whether there is a time $t \geq 0$ at which these two moving disks collide? (See Fig. 4(b).) (Would like a single mathematical expression, and not a program that simulates the motion over time.) Your test does *not* need to determine the first time of collision, just *whether* an intersection ever occurs.)

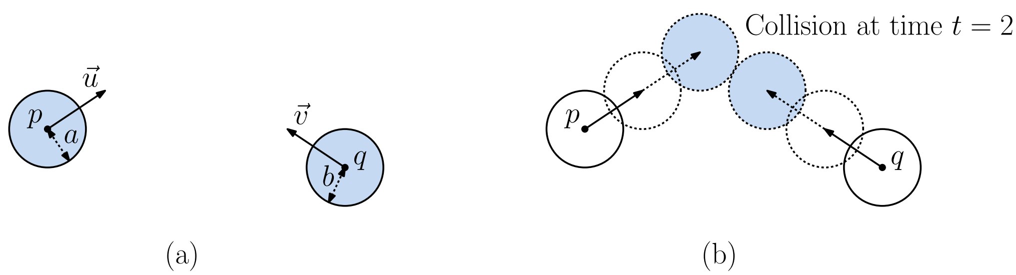

Figure 4: Challenge Problems: Disk collision detection.

(Hint: Start by writing an expression for the distance between the two center points as a function of time t.)

Submission Instructions: Because we will use GradeScope for grading, please write your solutions on the solution template, which can be downloaded from the class handouts page. (If there is insufficient space to write your answer, please note this within the space provided.)

We encourage you to submit through ELMS as a pdf file, but it is also acceptable to submit a hand-written document in class. If you submit through ELMS, you can either type your answer in Word and convert to a pdf document, or you can write your answer by hand and submit a scanned version. If you do the latter, please use an image-enhancing app like CamScanner. (Scanned homeworks that are not clearly legible may be returned to you, requiring rescanning and possibly recopying.)