In Class Exercise – Operational Semantics Derivations

Tuesday, September 25, 2018

This problem uses a subset of the language of arithmetic expressions we saw in lecture:

$$a ::= n \mid X \mid a + a$$

where $X \in \mathit{Var}$ ranges over variables, and a program state $\sigma : \mathit{Var} \to n$ maps variables to integers n.

1. Consider the following big-step semantics rules:

$$\frac{}{\langle n, \sigma\rangle \to n} \qquad \frac{}{\langle X, \sigma\rangle \to \sigma(X)} \qquad \frac{\langle a_1, \sigma\rangle \to n \qquad \langle a_2, \sigma\rangle \to m \qquad p = n + m}{\langle a_1 + a_2, \sigma\rangle \to p}$$

   Write a derivation showing that $\langle (1 + X) + 3, \sigma\rangle \to 6$ if $\sigma = [X \mapsto 2]$.

2. Consider the following small-step semantics rules:

$$\frac{}{X \to_\sigma \sigma(X)} \qquad \frac{a_1 \to_\sigma a_1'}{a_1 + a_2 \to_\sigma a_1' + a_2} \qquad \frac{a_2 \to_\sigma a_2'}{n + a_2 \to_\sigma n + a_2'} \qquad \frac{p = n + m}{n + m \to_\sigma p}$$

   Write a sequence of derivations showing that $(1 + X) + 3 \to_\sigma^* 6$ if $\sigma = [X \mapsto 2]$. Here $\to_\sigma^*$ is the reflexive, transitive closure of $\to_\sigma$.